Diagnosing Heating and Energy Transfer in Collisionless Kinetic Plasmas

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Collaborators

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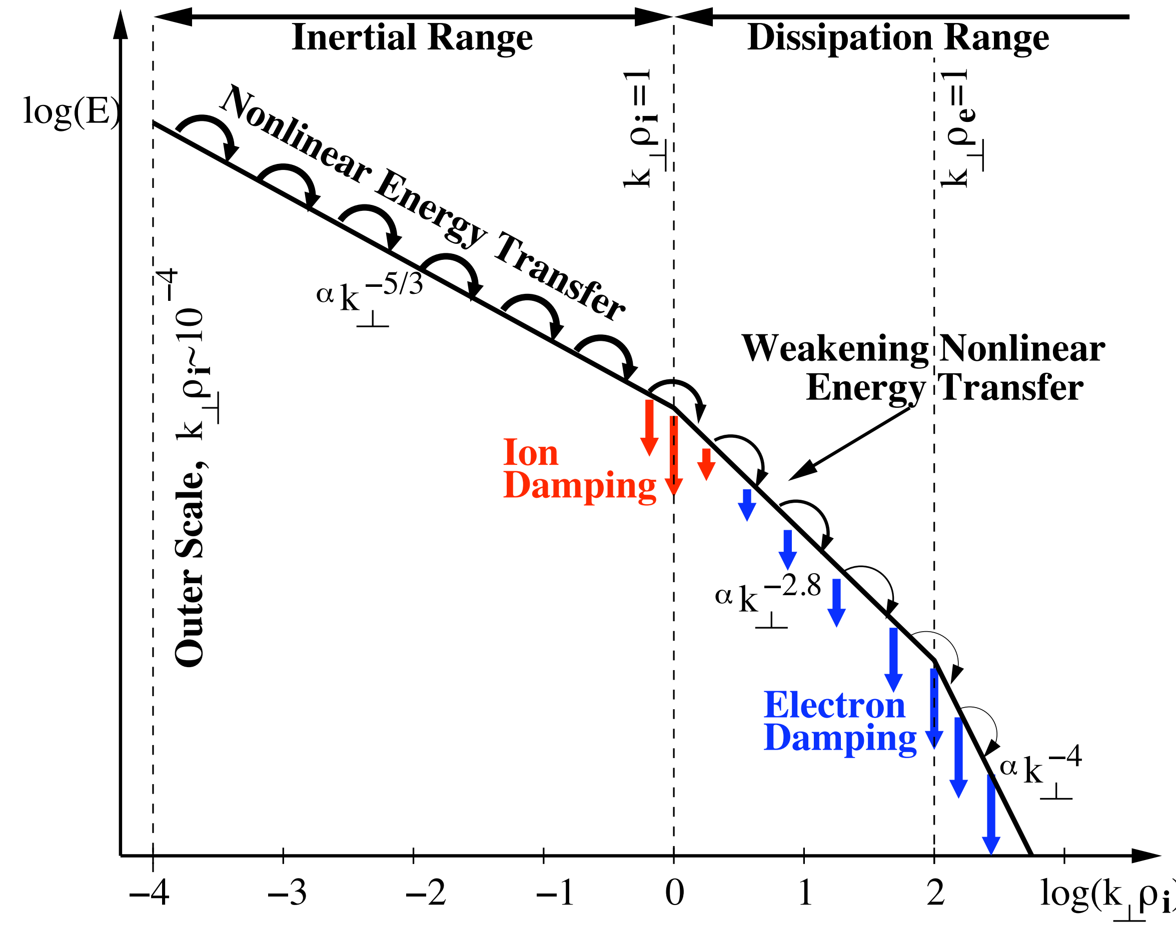
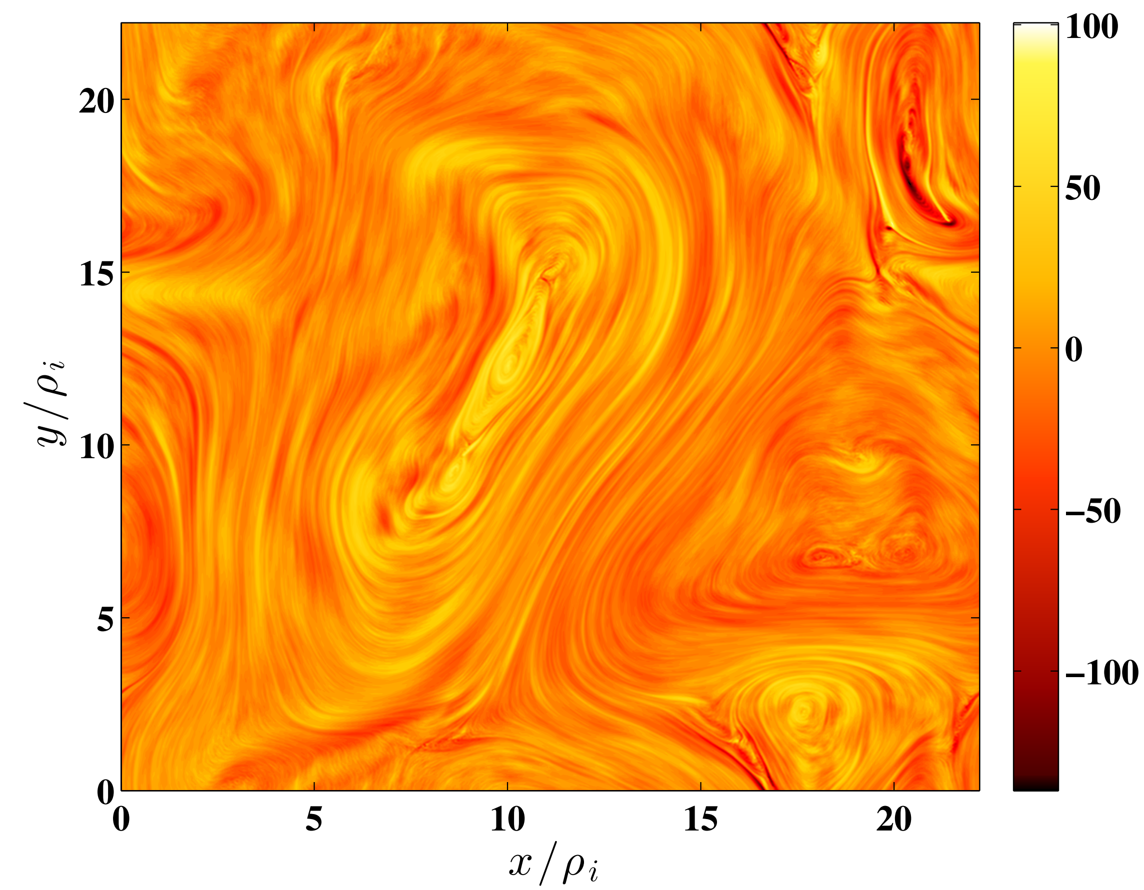
Particle energization in turbulence

- Nonlinear cascade of MHD Alfvén waves transitions to a cascade of kinetic Alfvén waves at the ion Larmor radius.

- Most dissipation begins at ion kinetic scales and includes (but is not limited to):
  - Wave-particle interactions (Landau, transit-time, cyclotron, …).
  - Current sheets also reconnect at ion scales and may be responsible for dissipation.

- Can we identify the energization mechanism?





Particle energization in turbulence

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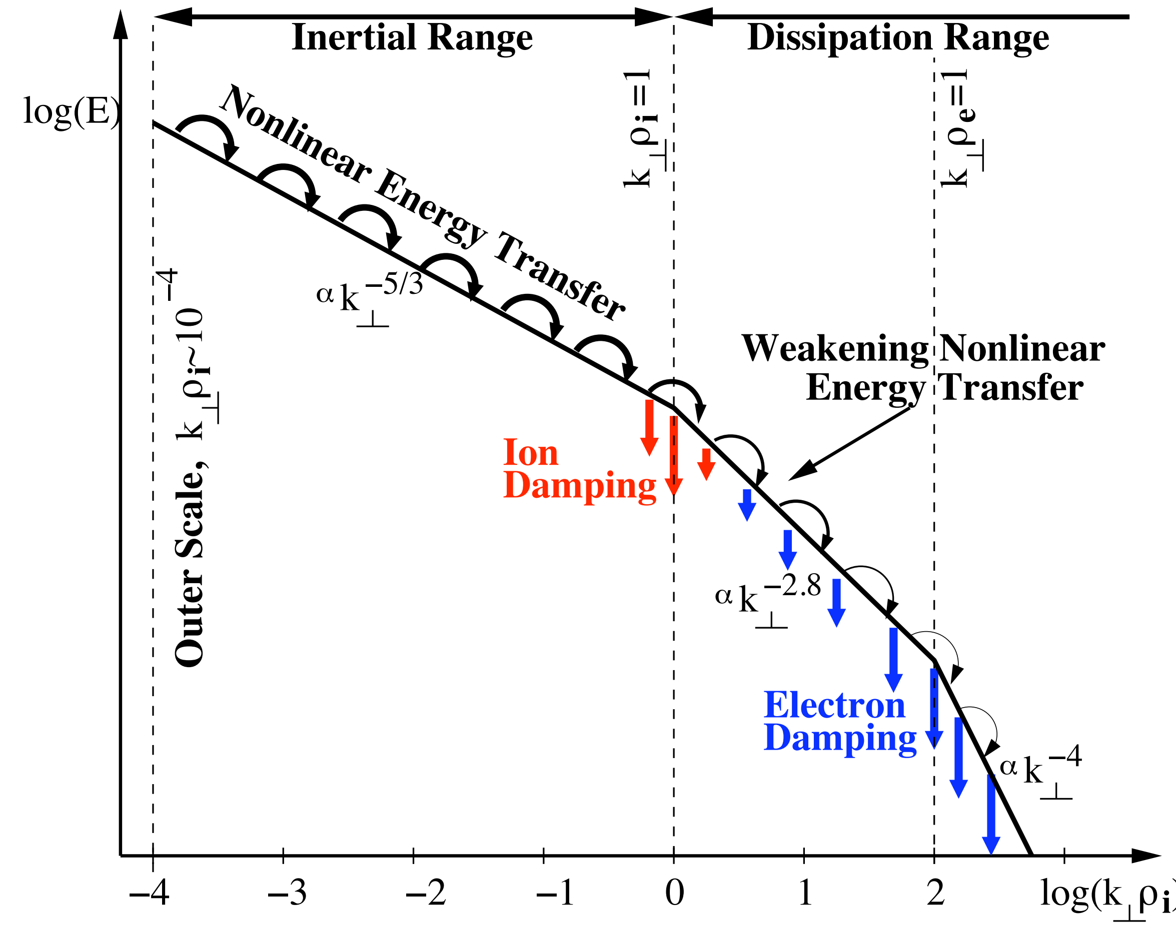

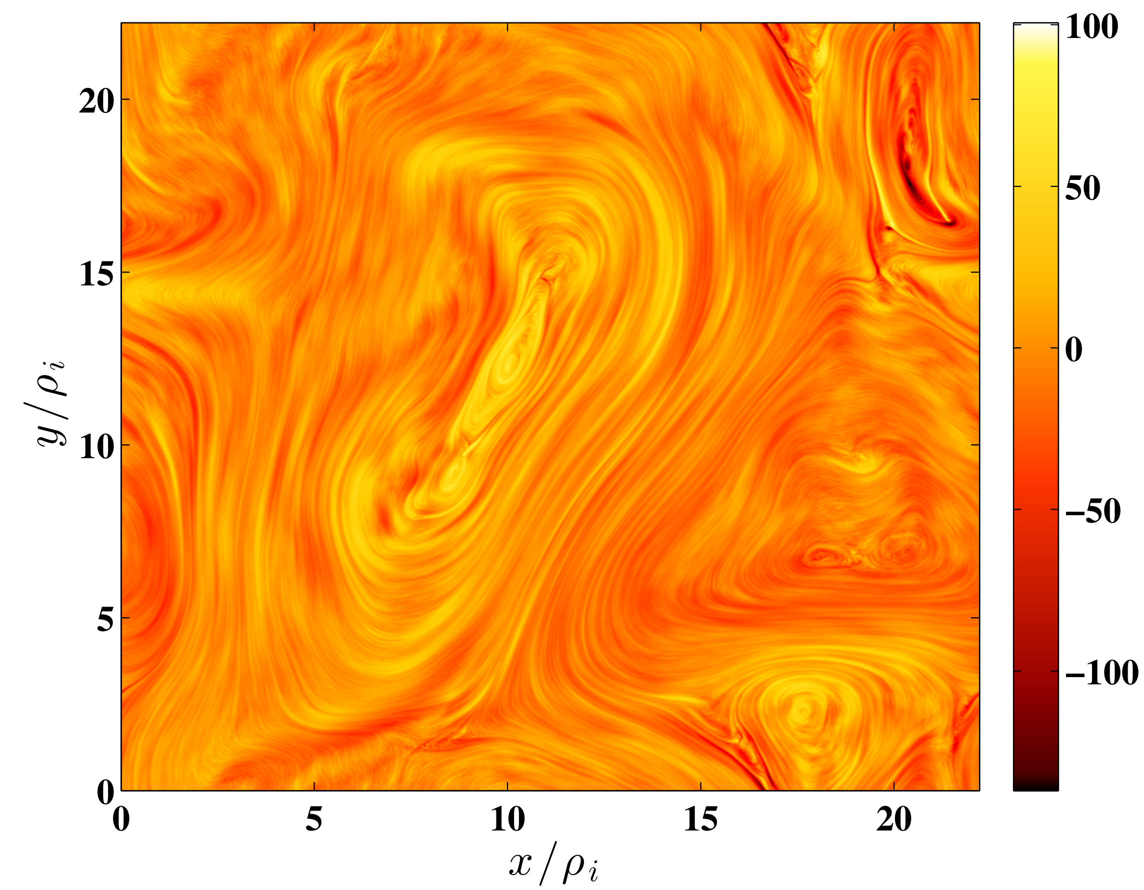

- Introduce the field particle correlation
  - Langmuir wave damping
  - Sod shock
  - Turbulence: Driven gyrokinetic and hybrid kinetic

- Introduce PiD
  - Langmuir wave damping
  - Sod shock

Simulation codes

Gkeyll [Juno et al 2018] Continuum Vlasov-Maxwell	***Pegasus*** [Kunz et al 2014] Hybrid particle-in-cell	***AstroGK*** [Numata et al 2010] Continuum gyrokinetics
Fully kinetic Eulerian ions and electrons	Kinetic Lagrangian ions and massless, isothermal fluid electrons	Gyrokinetic Eulerian ions and electrons —$\omega \ll \Omega_{cs}$, $k_{\parallel} \ll k_{\perp}$, & $\delta f \ll f_0$
Up to 3x3v ions and electrons	Up to 3x3v ions and 3x electrons	Up to 3x2v ions and electrons
Non-relativistic	Non-relativistic	Non-relativistic
	Quasineutral	Quasineutral
		Cyclotron waves and entire fast/whistler mode branch ordered out
Collisionless	Collisionless ions, hyper-resistivity added to electrons	Includes Landau collision operator
		Background quantities not evolved

Field Particle Correlations

Vlasov-Poisson [Howes, Klein, & Li (2017)]

$$\frac{\partial f_s}{\partial t} + v\frac{\partial f_s}{\partial x} - \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_s}{\partial v} = 0$$

$$f_s(x,v,t) = f_{s0}(v) + \delta f_s(x,v,t)$$

Separation useful in some cases but not necessary

$$\frac{\partial \delta f_s}{\partial t} = -v\frac{\partial \delta f_s}{\partial x} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_{s0}}{\partial v} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial \delta f_s}{\partial v}$$

Multiply by $mv^2/2$ and integrate to obtain the energy equation

$$\frac{\partial W_s}{\partial t} = \int dx \int dv \frac{1}{2}m_s v^2 \left[-v\frac{\partial \delta f_s}{\partial x} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_{s0}}{\partial v} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial \delta f_s}{\partial v}\right]$$

$$\frac{\partial W_s}{\partial t} = \int dx \int dv\ \frac{1}{2}m_s v^2 \frac{\partial f_s}{\partial t}$$

Vlasov-Poisson [Howes, Klein, & Li (2017)]

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$$\frac{\partial W_s}{\partial t} = \int dx \int dv\ \frac{1}{2}m_s v^2 \frac{\partial f_s}{\partial t}$$

In terms of fluid moments

$$\frac{1}{2}\frac{\partial P}{\partial t} = -\nabla \cdot \frac{\mathbf{Q}}{2} + qn\mathbf{u}\cdot\mathbf{E}$$

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$$\frac{\partial W_s}{\partial t} = \int dx \int dv \frac{1}{2} m_s v^2 \left[\cancel{-v\frac{\partial \delta f_s}{\partial x}} + \cancel{\frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_{s0}}{\partial v}} + \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial \delta f_s}{\partial v} \right]$$

$$\frac{\partial W_s}{\partial t} = -\int dx \int dv \; q_s \frac{v^2}{2}\frac{\partial \delta f_s(x,v,t)}{\partial v} E(x,t) = \int dx \int dv \; q_s v \delta f_s(x,v,t) E(x,t)$$

$$\frac{\partial W_s}{\partial t} = -\int dx \frac{\partial \phi}{\partial x} \int dv \; q_s v \delta f_s = \int dx \; j_s E$$

Vlasov-Poisson [Howes, Klein, & Li (2017)]

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The field particle correlation: $C_1(v,t,\tau) = C_\tau \left(-q_s \frac{v^2}{2} \frac{\partial \delta f_s(x_0,v,t)}{\partial v}, E(x_0,t) \right)$

Alternative for cases in which df/dv is difficult to compute

$$C_2(v,t,\tau) = C_\tau \left(q_s v \delta f_s(x_0,v,t), E(x_0,t) \right)$$

In discrete form

$$C_1(v,t_i,\tau) = \frac{1}{N} \sum_{j=i}^{i+N} q_s \frac{v^2}{2} \frac{\partial \delta f_{sj}(v)}{\partial v} E_j$$

Note that f or δf can be used

$$t_j \equiv t(j\Delta t) \qquad \tau = N\Delta t,$$

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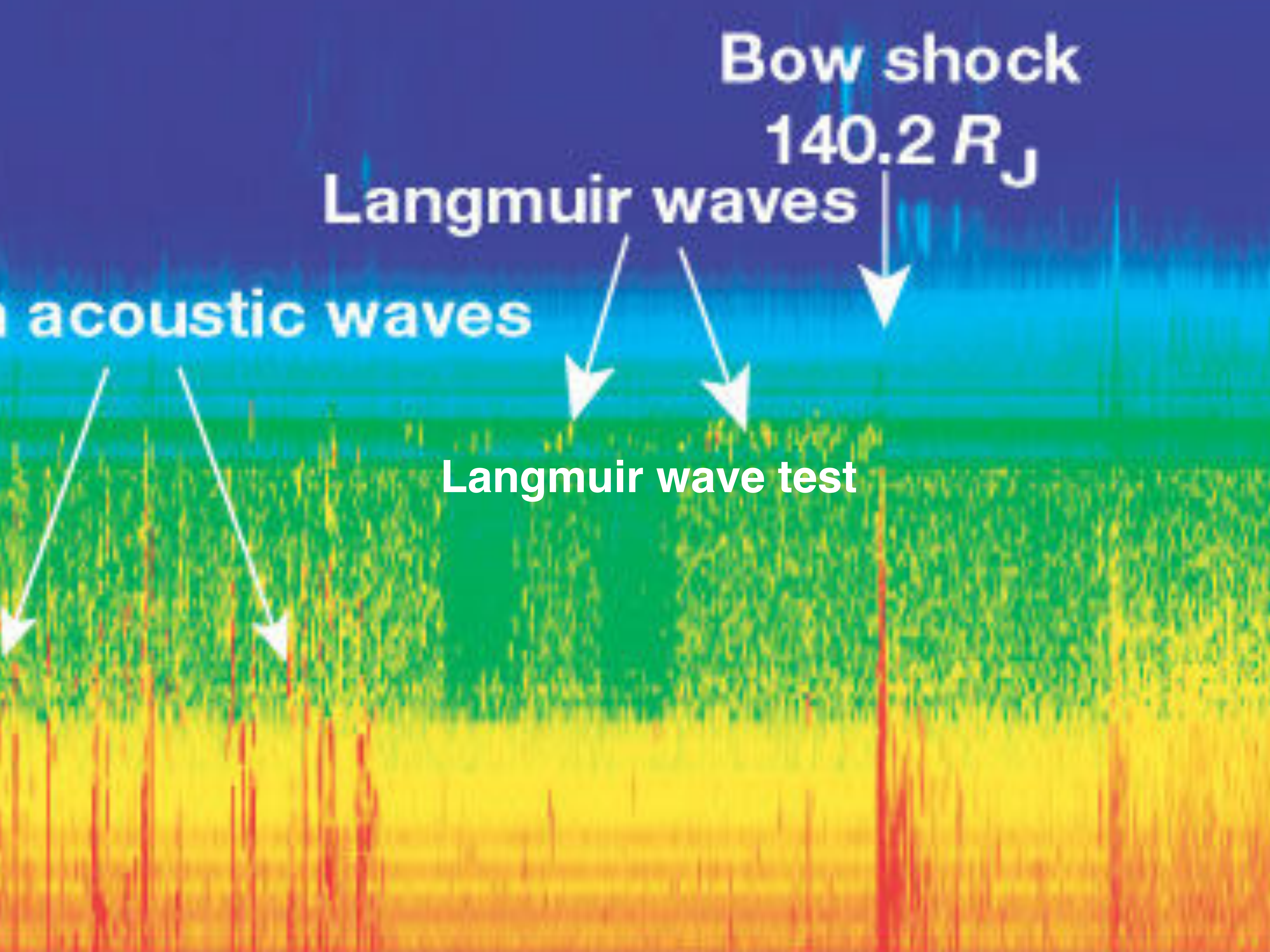
Other quantities that will appear:

$$W_s = \int w_s dx \qquad \Delta w_s = \int_0^t (\partial w / \partial t') dt' \qquad R = \frac{\int_{v_1}^{v_2} |C(v)| dv}{\int_{-v_{max}}^{v_{max}} |C(v)| dv}$$

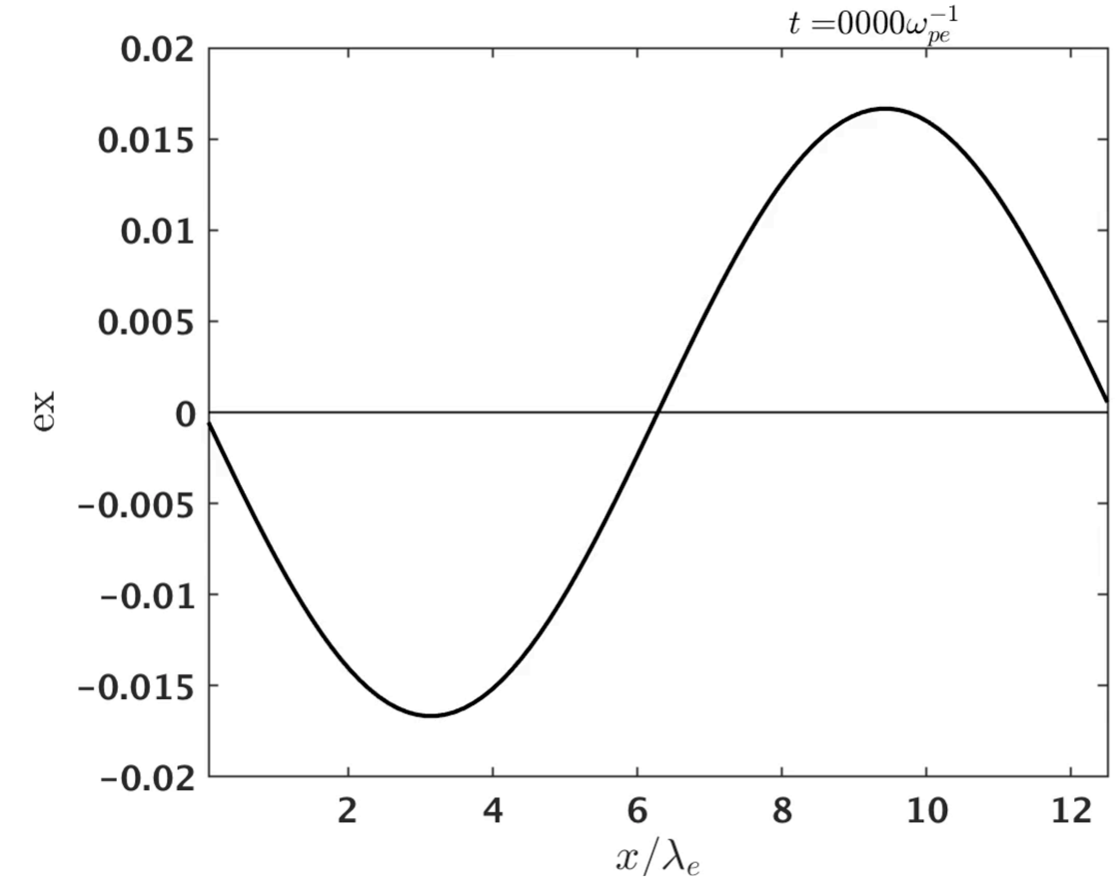
$$< C_1 >= \int_0^t C_1(v,t',\tau) dt'$$

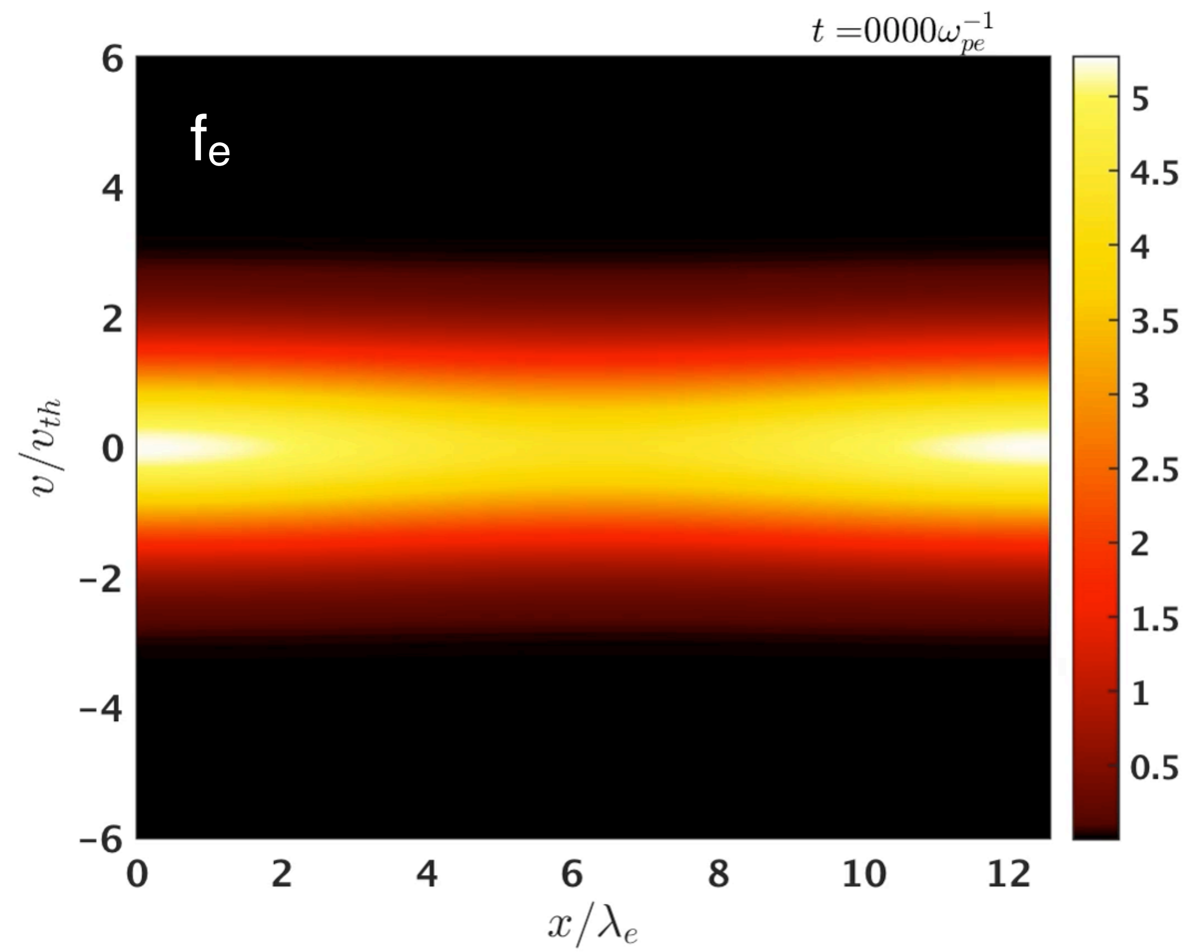

Bow shock
140.2 R_J
Langmuir waves
acoustic waves
Langmuir wave test

Langmuir wave setup (Gkeyll)

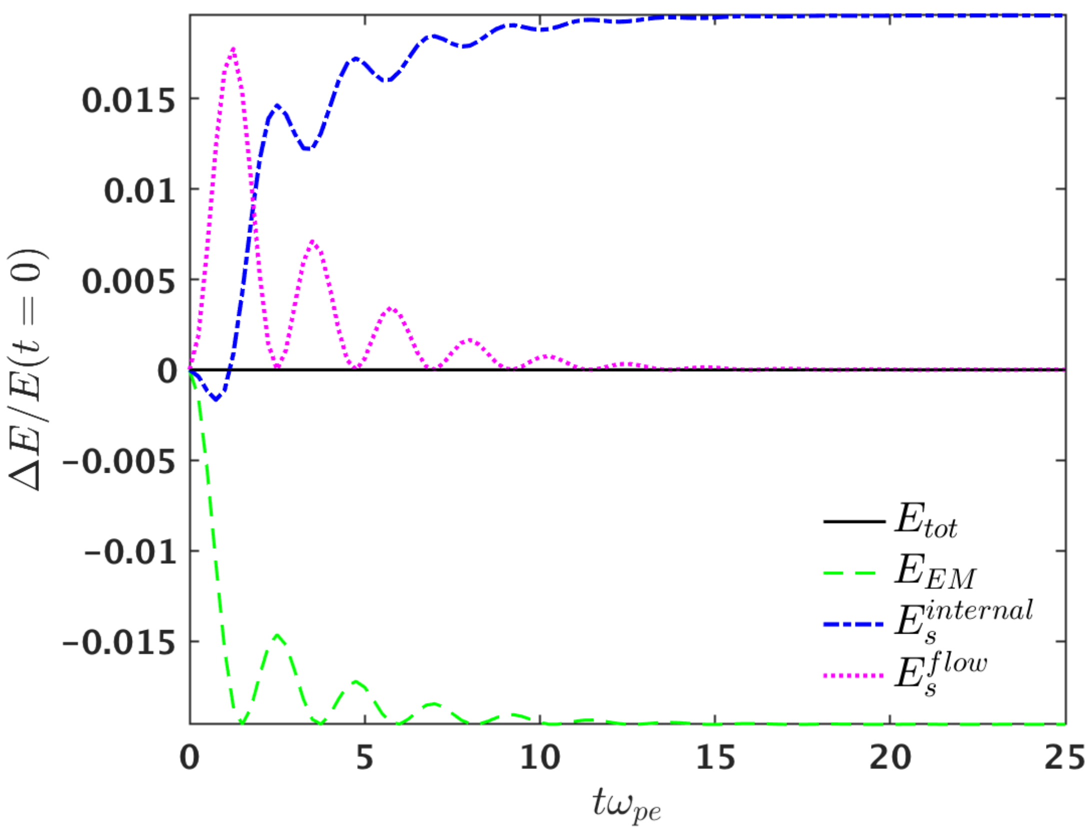
$$m_p/m_e = 1836$$
$$T_p/T_e = 1$$
$$v_{te}/c = 1/12$$
$$n = 1 + \alpha\cos(kx),\ \alpha = 0.1$$
$$k\lambda_D = 0.5$$
$$L_x = 4\pi\lambda_D,\ n_x = 32\text{cells} \rightarrow 96\text{nodes}$$
$$v_{ex} \in [-6,6]v_{te},\ n_{vx} = 128\text{cells} \rightarrow 378\text{nodes}$$
$$v_{px} \in [-6,6]v_{tp},\ n_{vx} = 16\text{cells} \rightarrow 48\text{nodes}$$

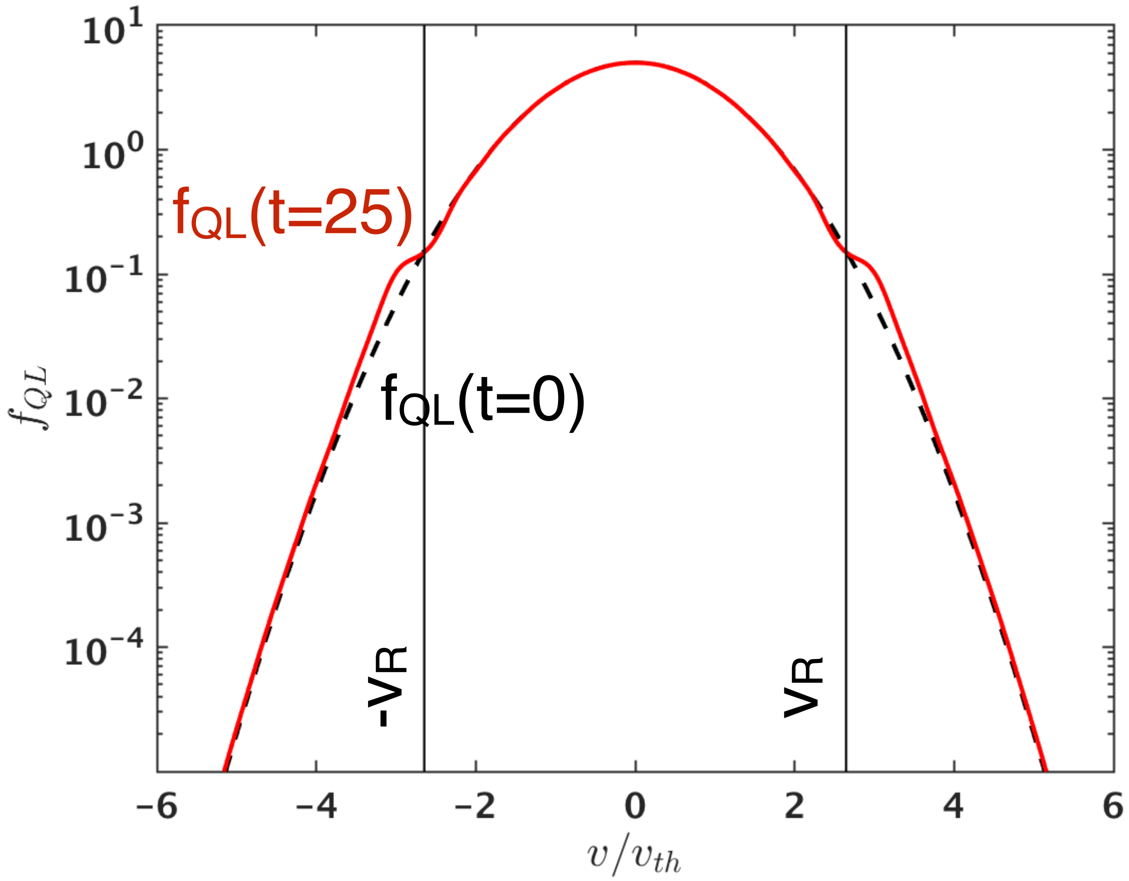
Initializes a standing Langmuir wave that is moderately damped, $-\gamma/\omega \simeq 0.1$





Langmuir wave result (Gkeyll)





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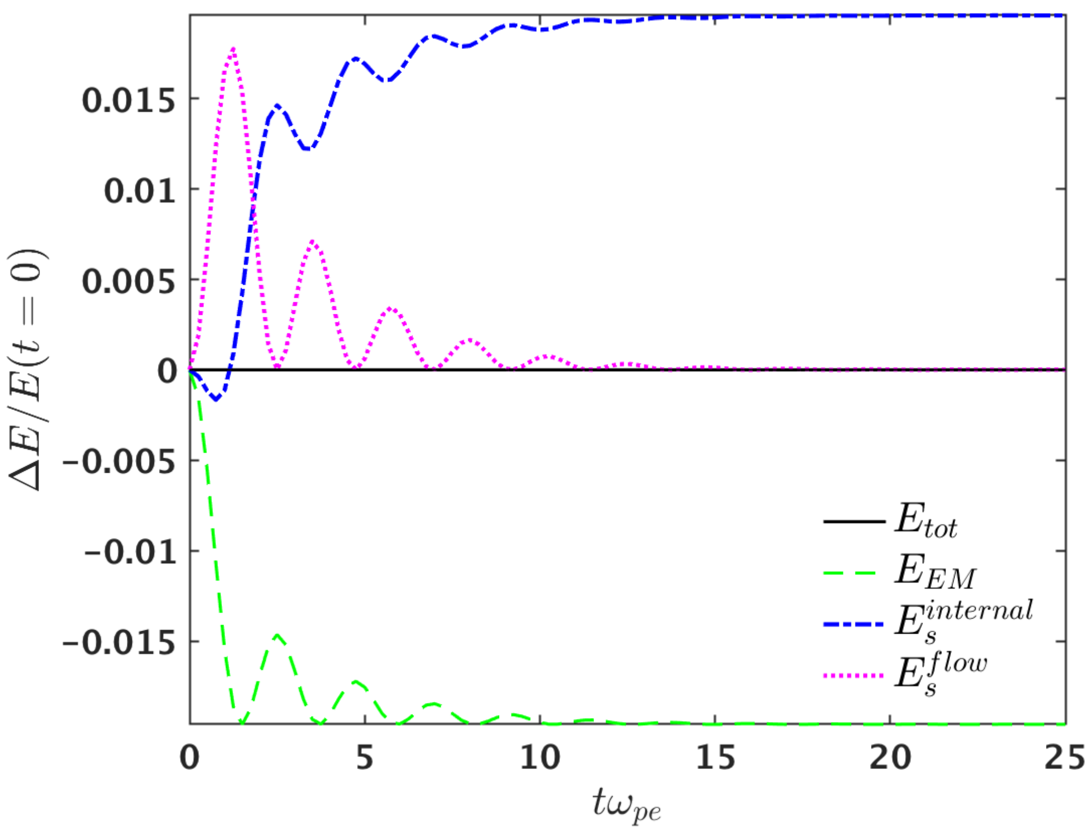
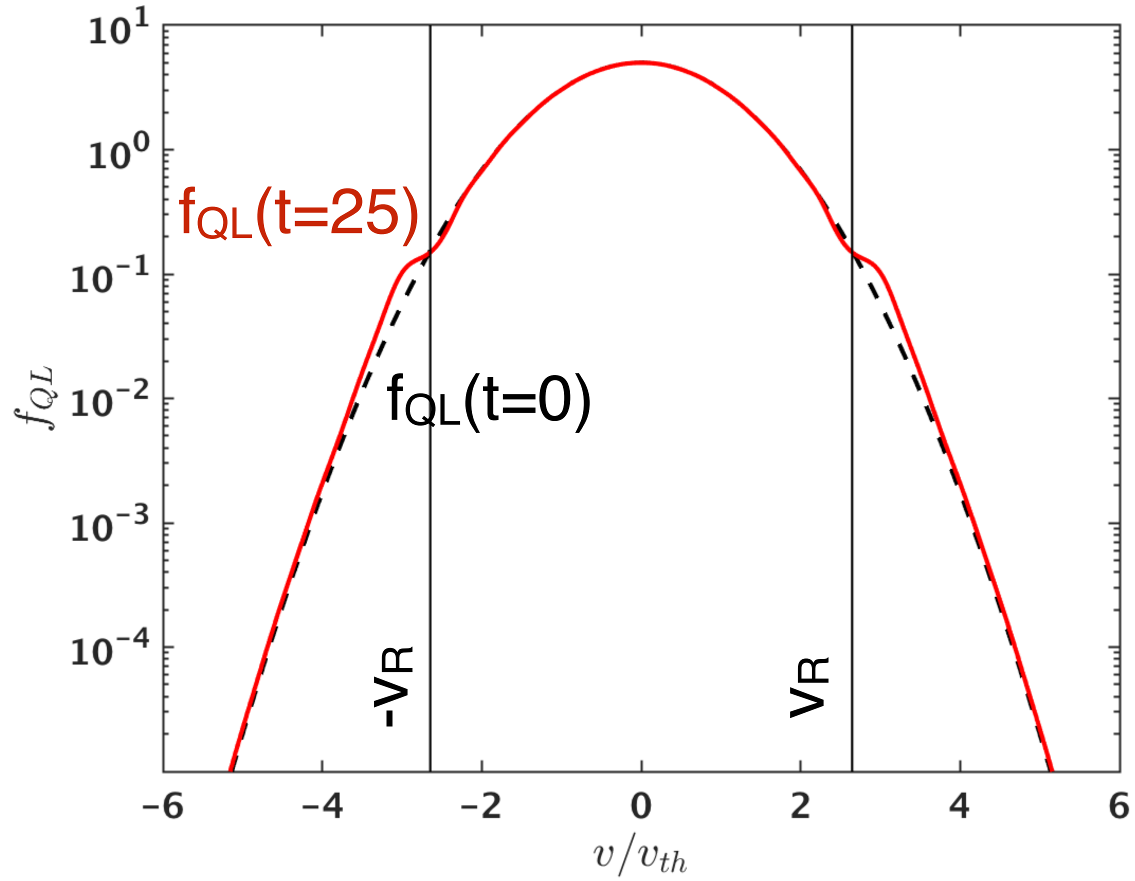
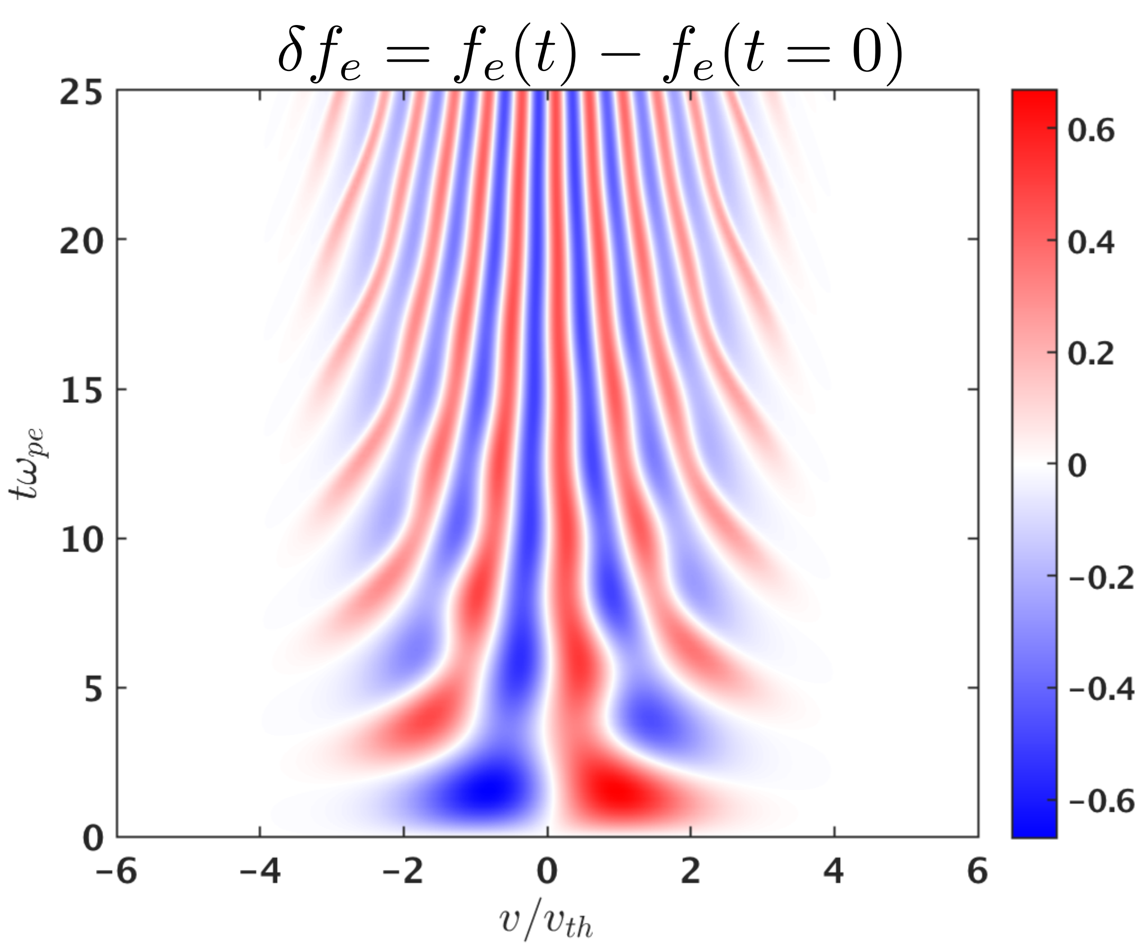
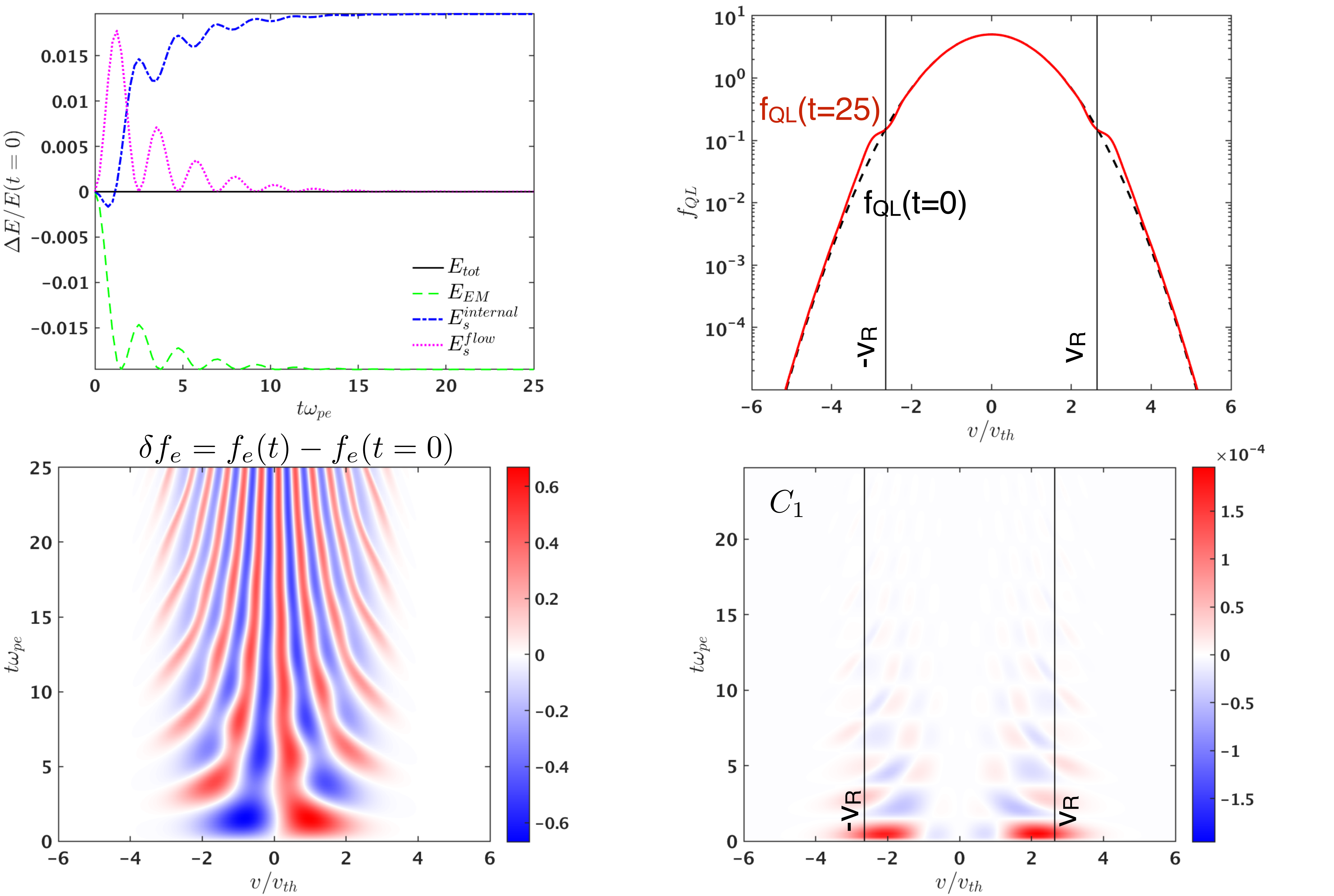




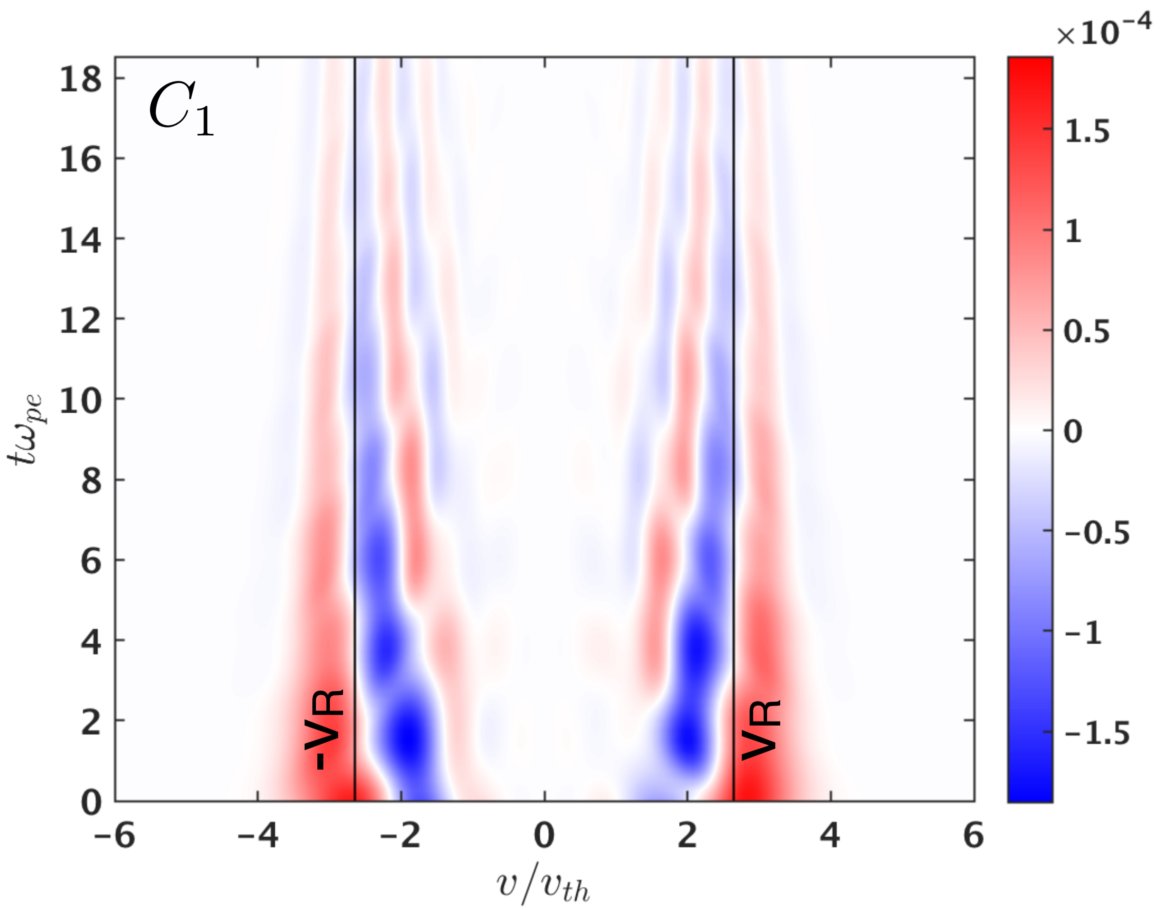


Langmuir wave result (Gkeyll), $\tau\omega_{pe}$=0



Langmuir wave result (Gkeyll), $\tau\omega_{pe}$=6.2

Now, we consider the field particle correlation with a finite time interval, $\tau\omega_{pe}$=6.2, which is ~1.5 times the period of the mode of interest



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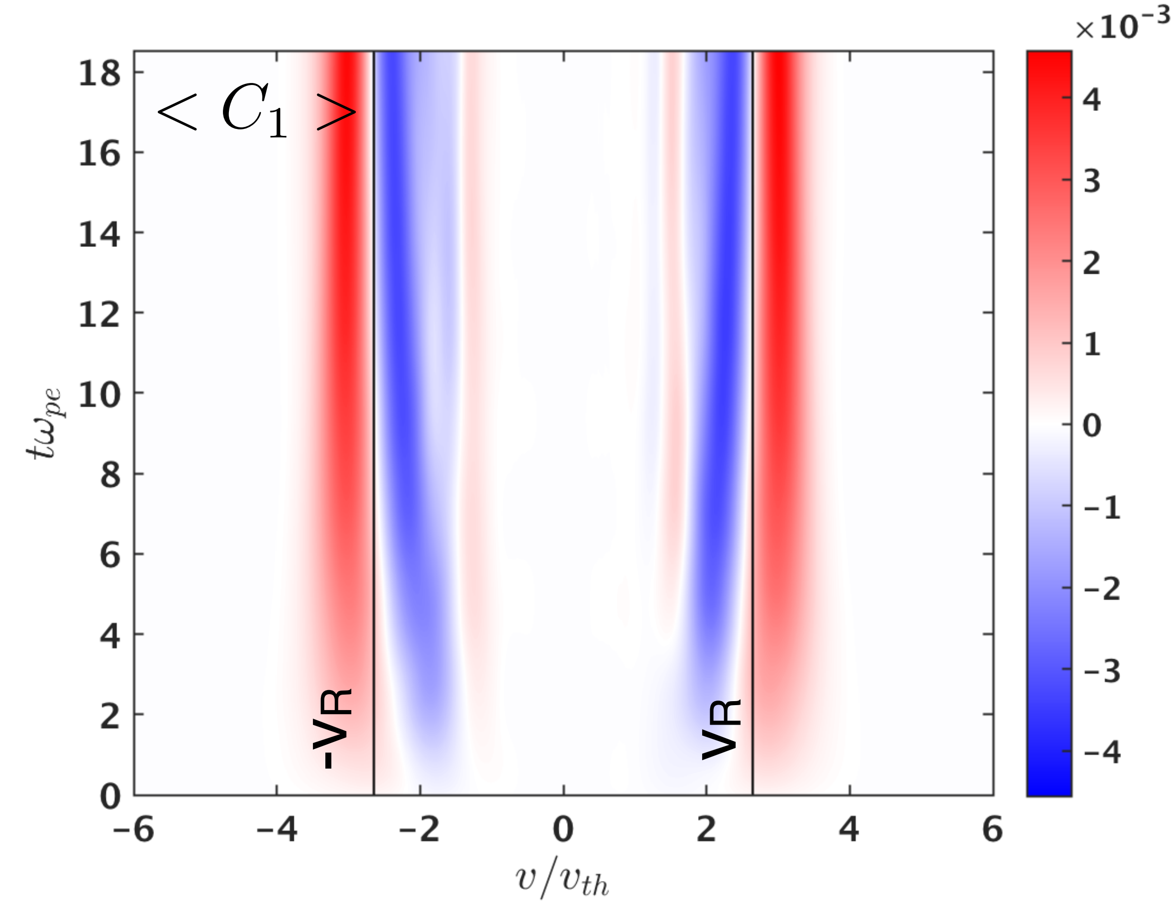
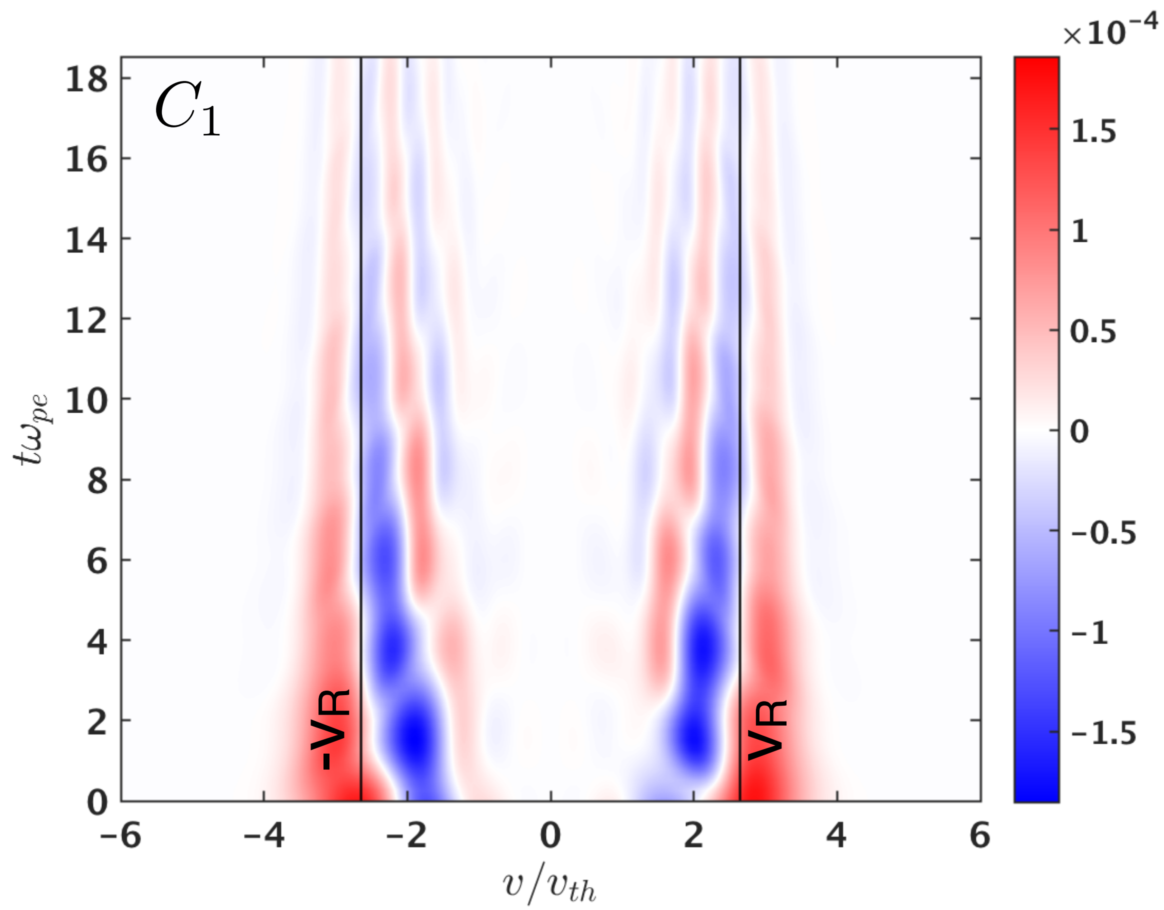
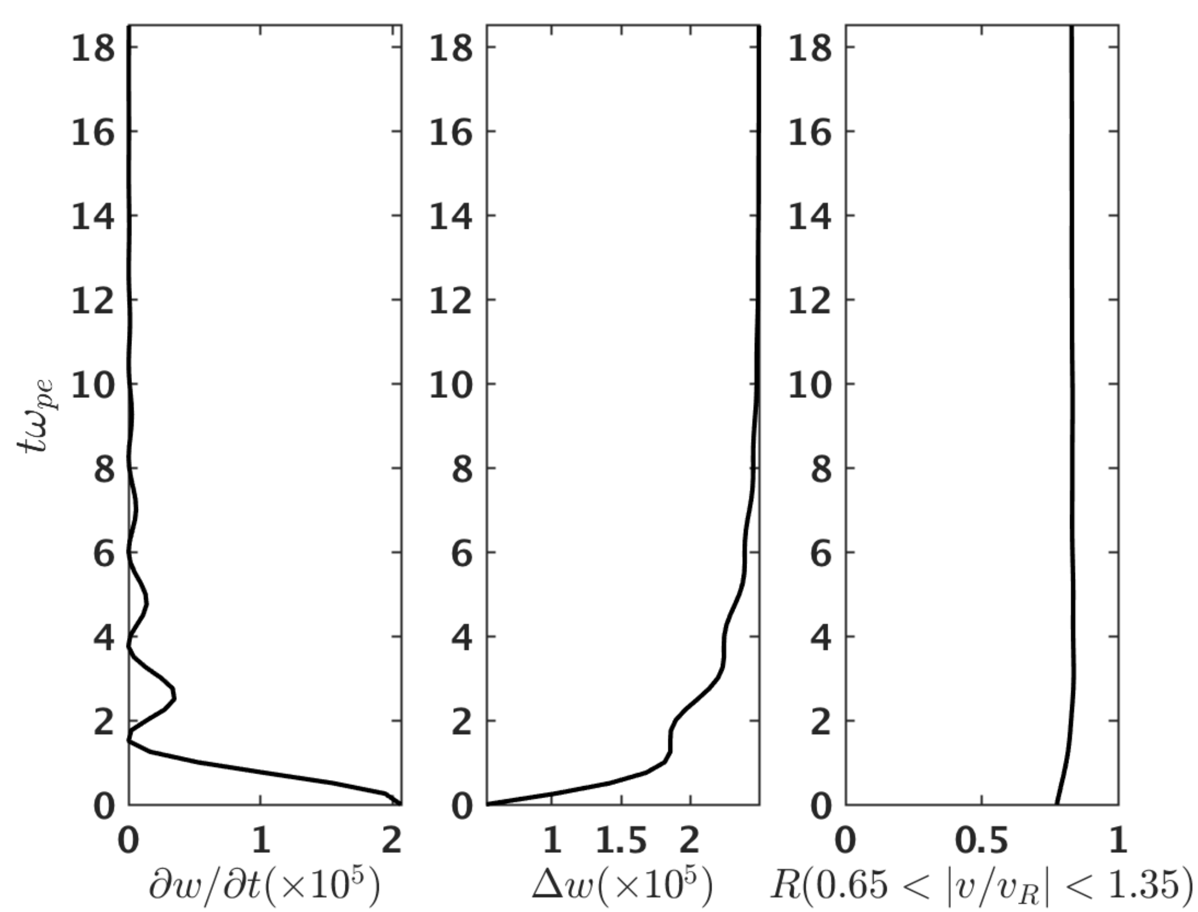
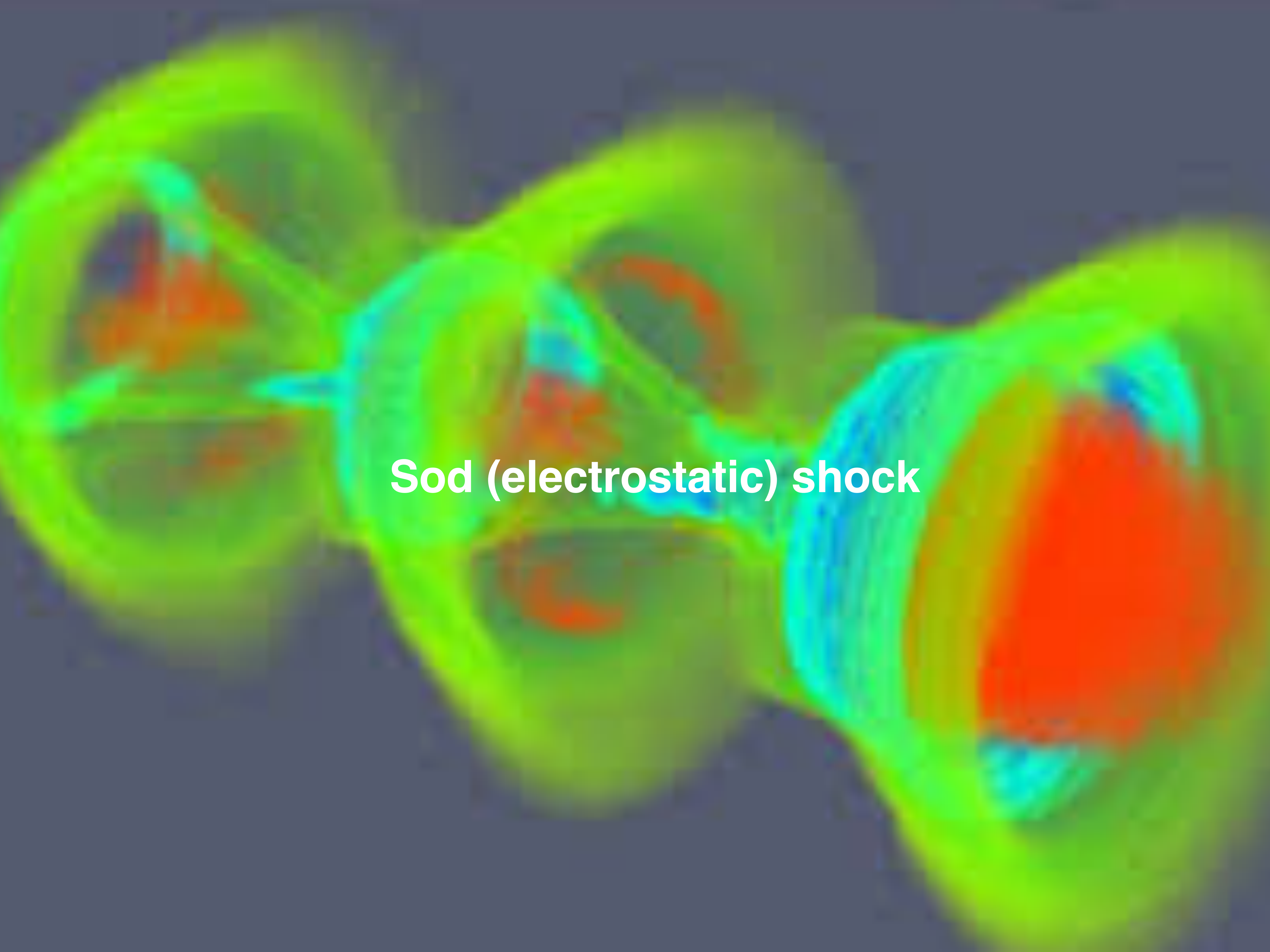


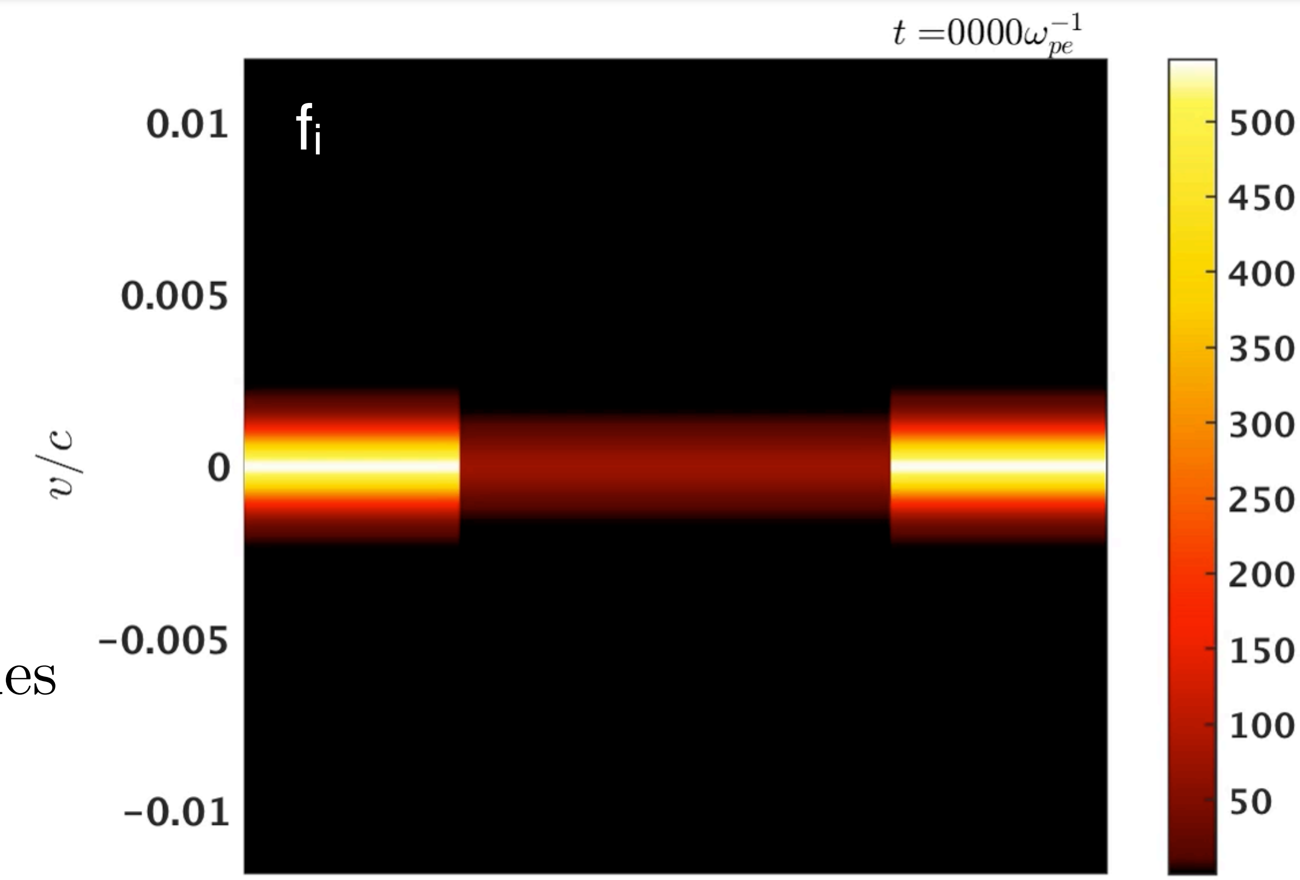


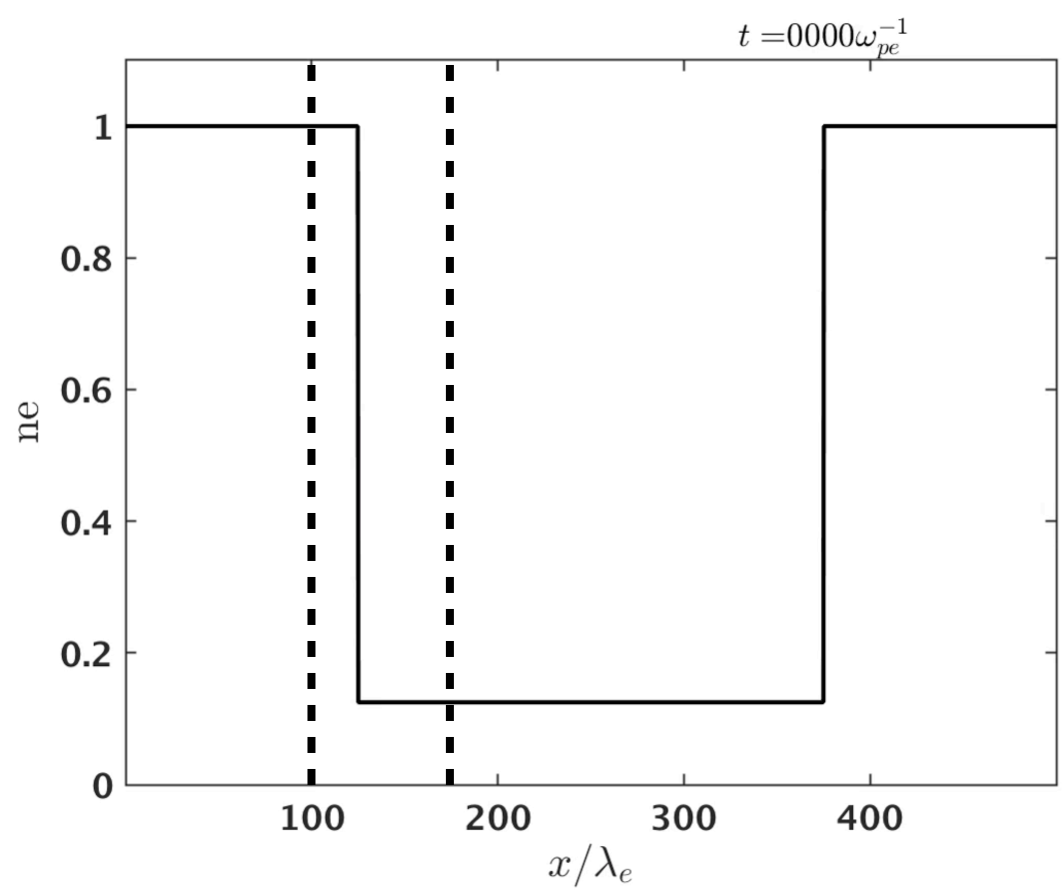


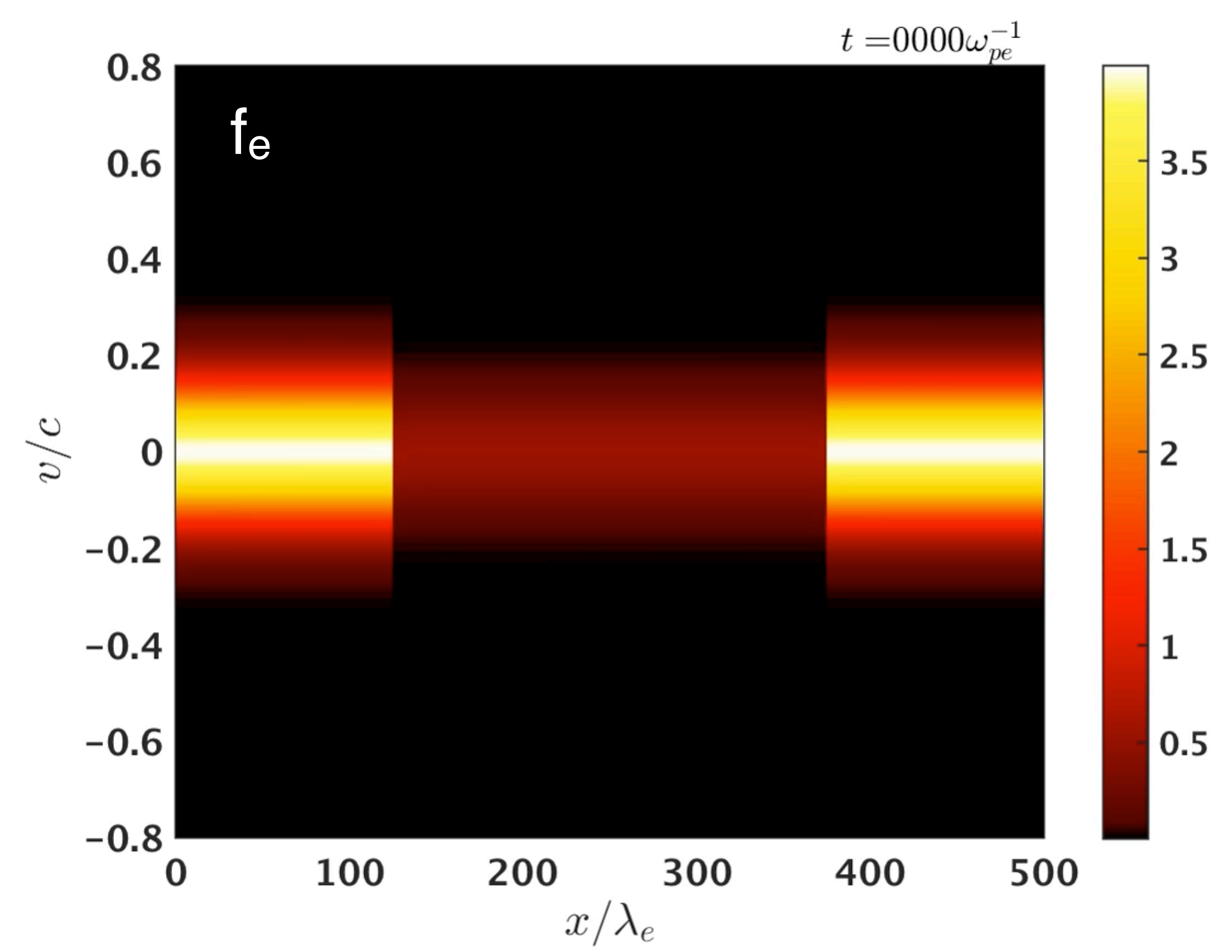

Sod (electrostatic) shock

Sod shock setup and evolution (Gkeyll)

$$m_p/m_e = 1836,\ v_{teL}/c = 0.1$$
$$n_L = 1,\ n_R = 0.125$$
$$T_{eL} = 0.01,\ T_{eR} = 0.008$$
$$T_{pL} = 0.001,\ T_{pR} = 0.0008$$
$$L_x = 500\lambda_D,\ n_x = 512\text{cells} \to 1536\text{nodes}$$
$$v_{ex} \in [-8, 8]v_{teL},\ n_{vx} = 256\text{cells} \to 768\text{nodes}$$
$$v_{px} \in [-16, 16]v_{tpL},\ n_{vx} = 1024\text{cells} \to 3072\text{nodes}$$







Sod shock setup and energy(Gkeyll)

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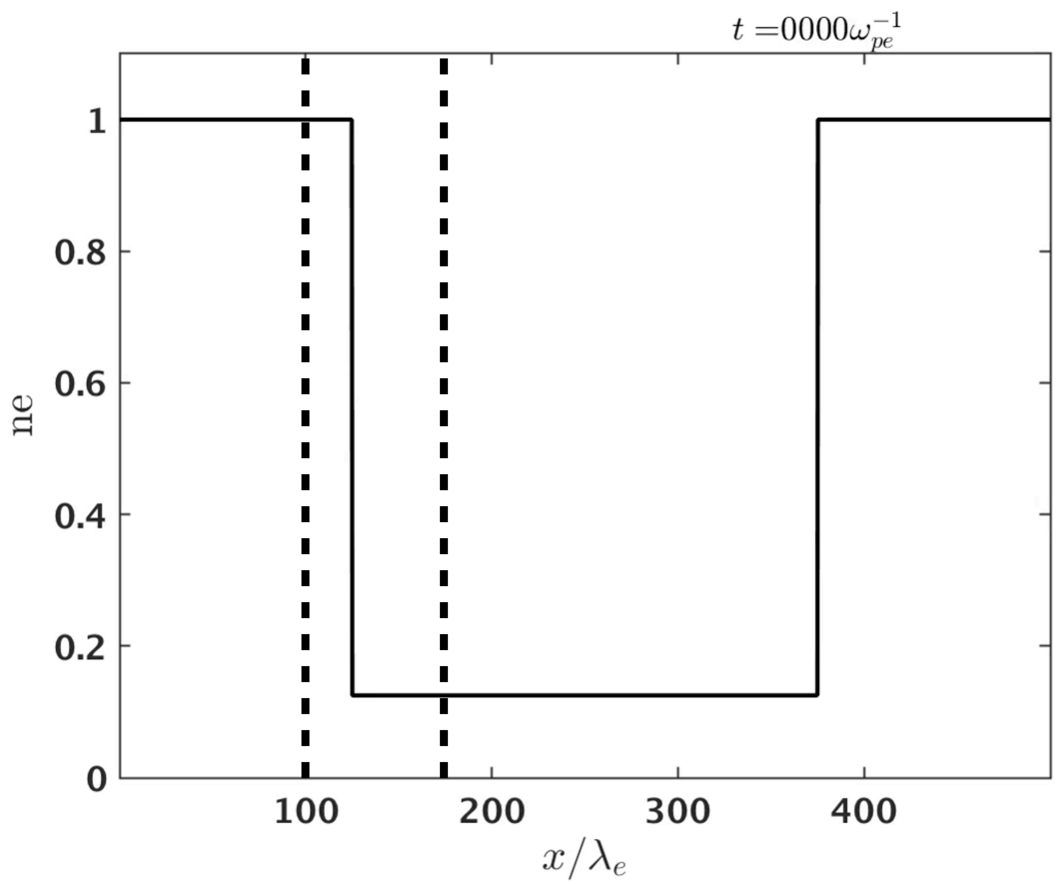
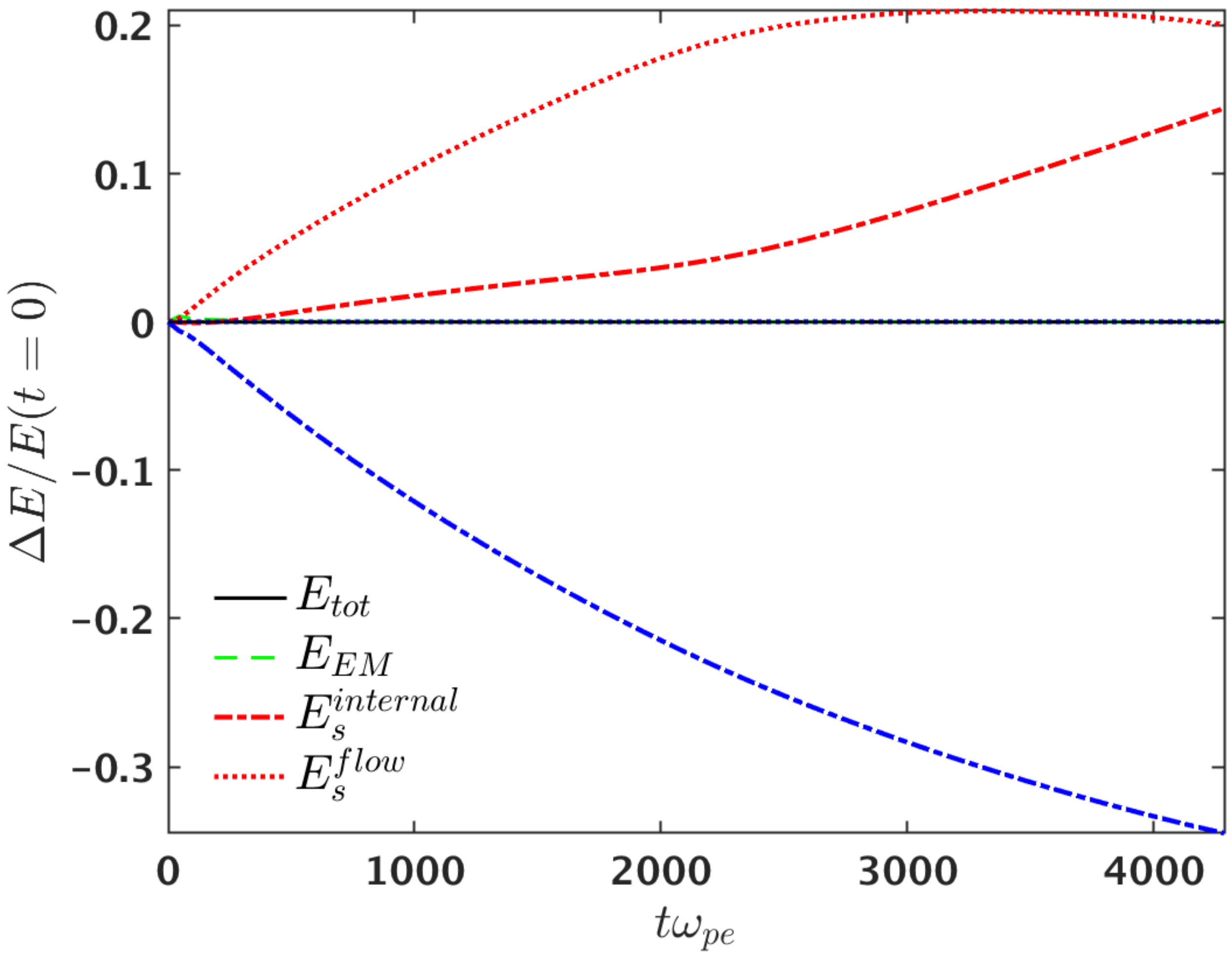
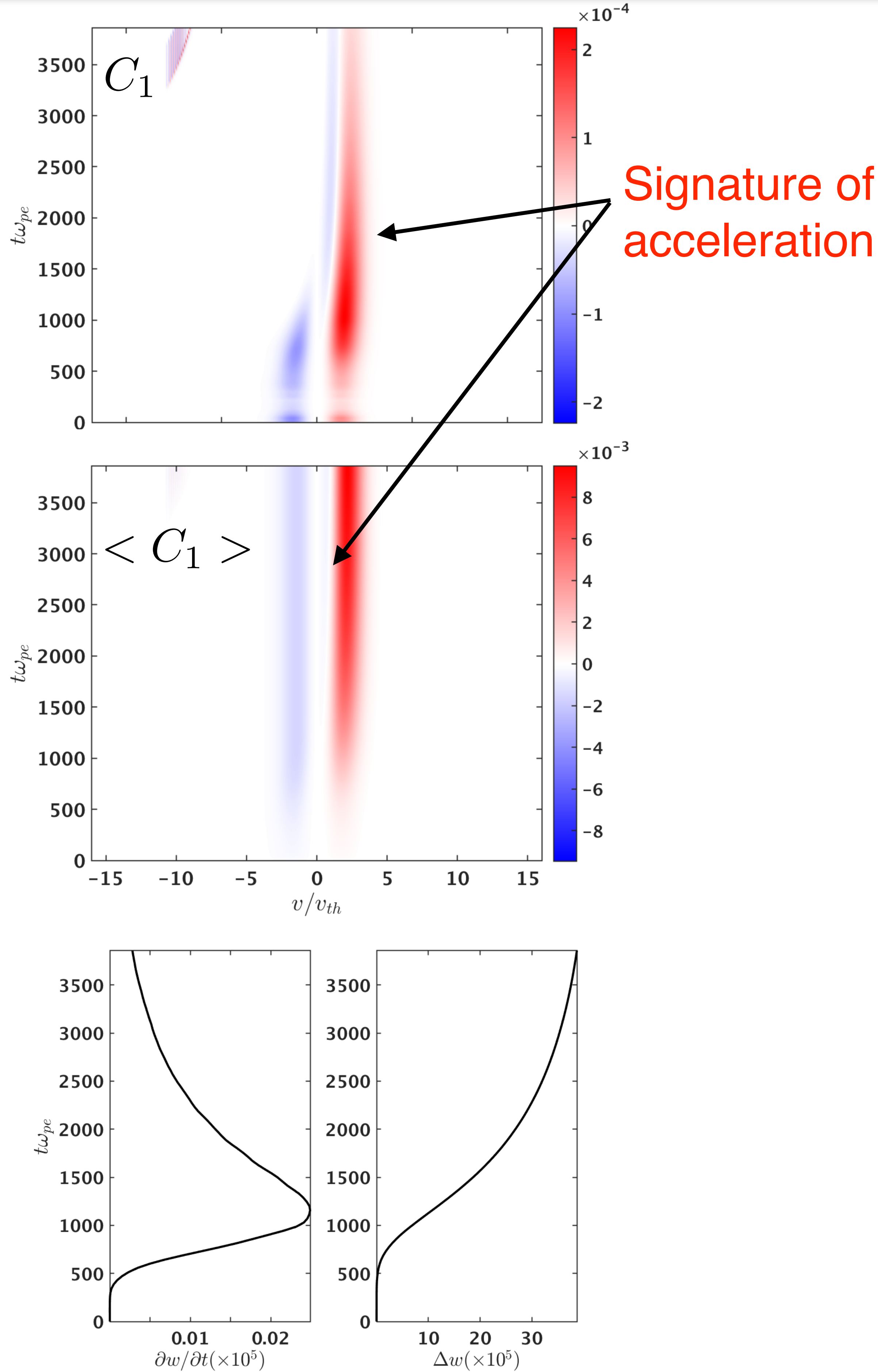
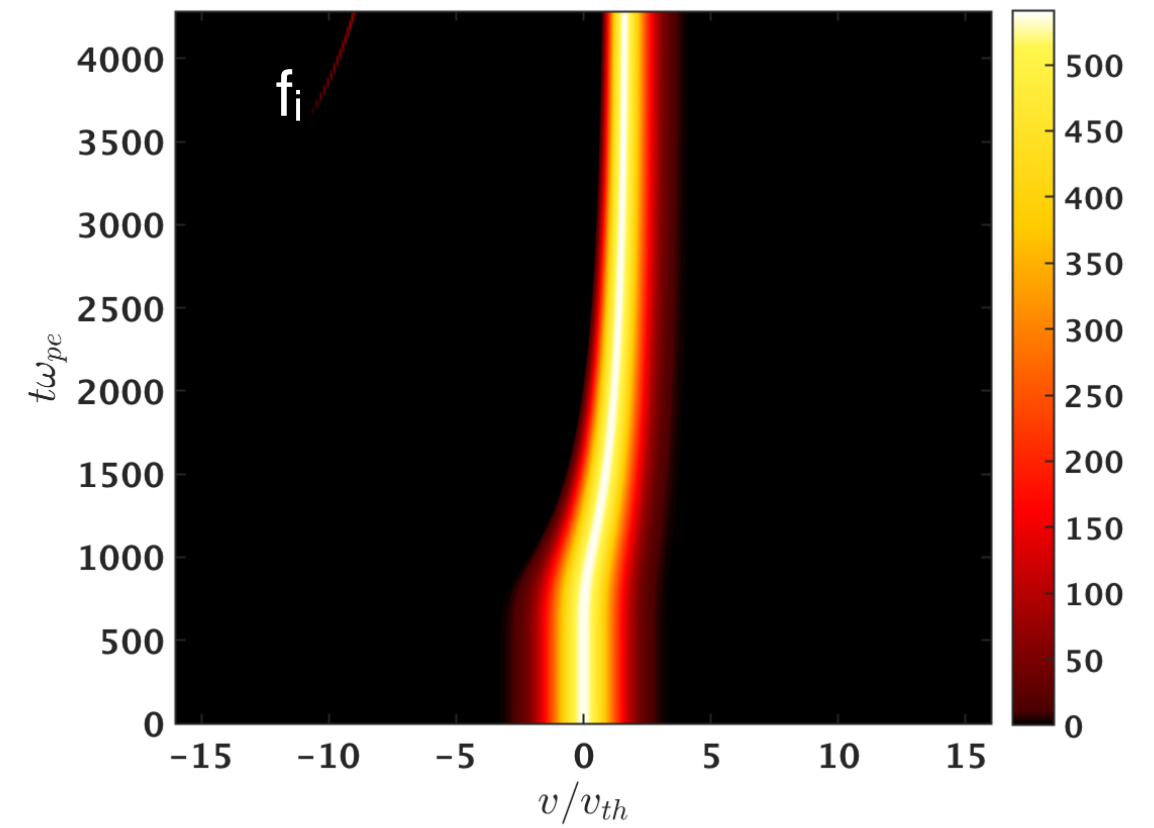




Proton shock result at x=100λ_e, $\tau\omega_{pe}$=386 (Gkeyll)





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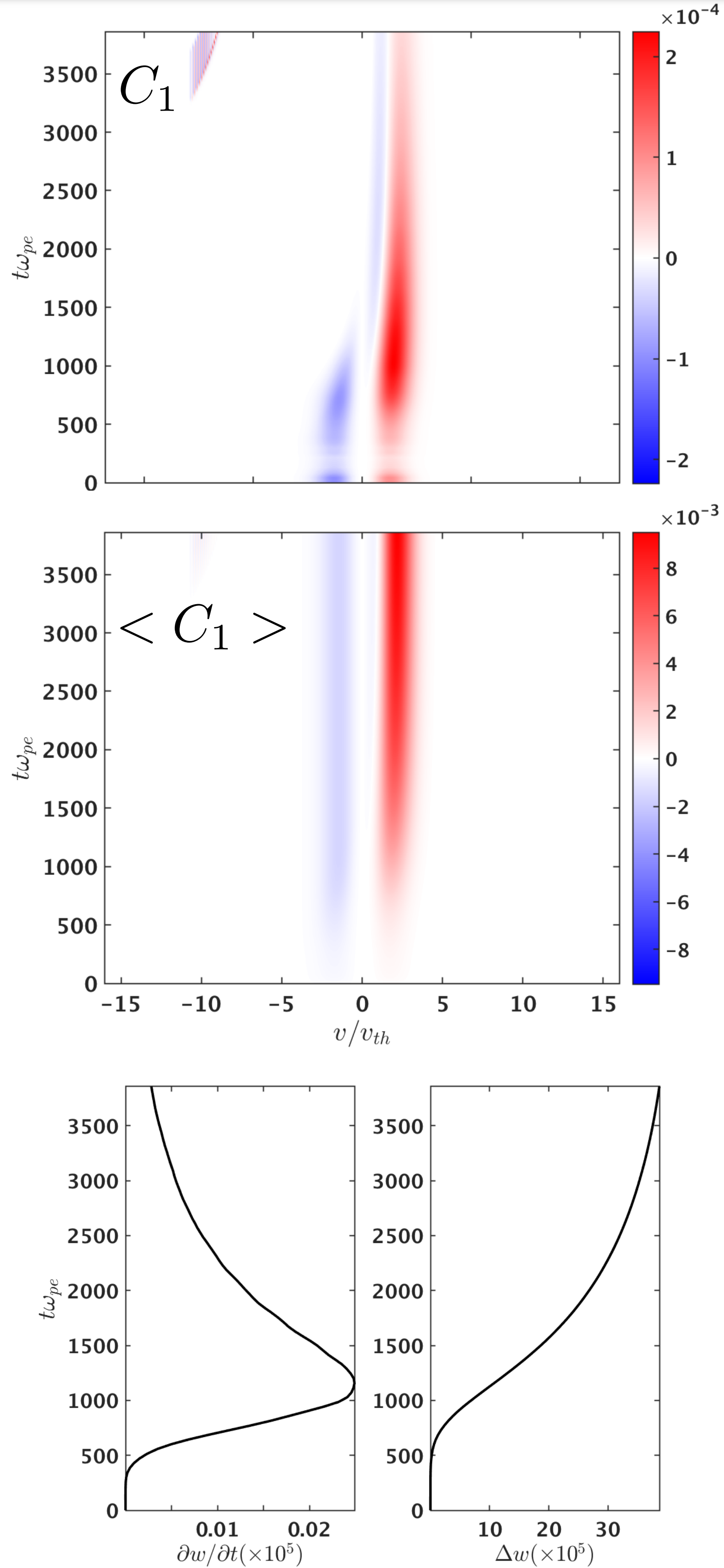

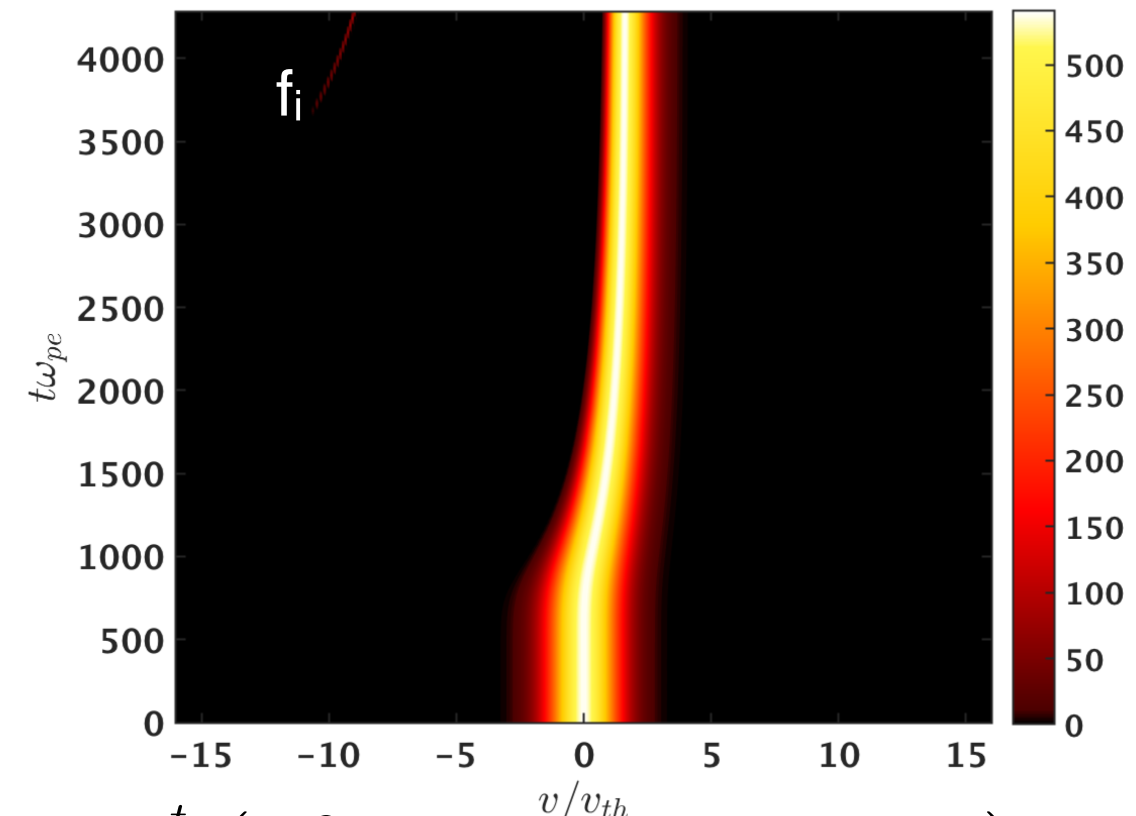

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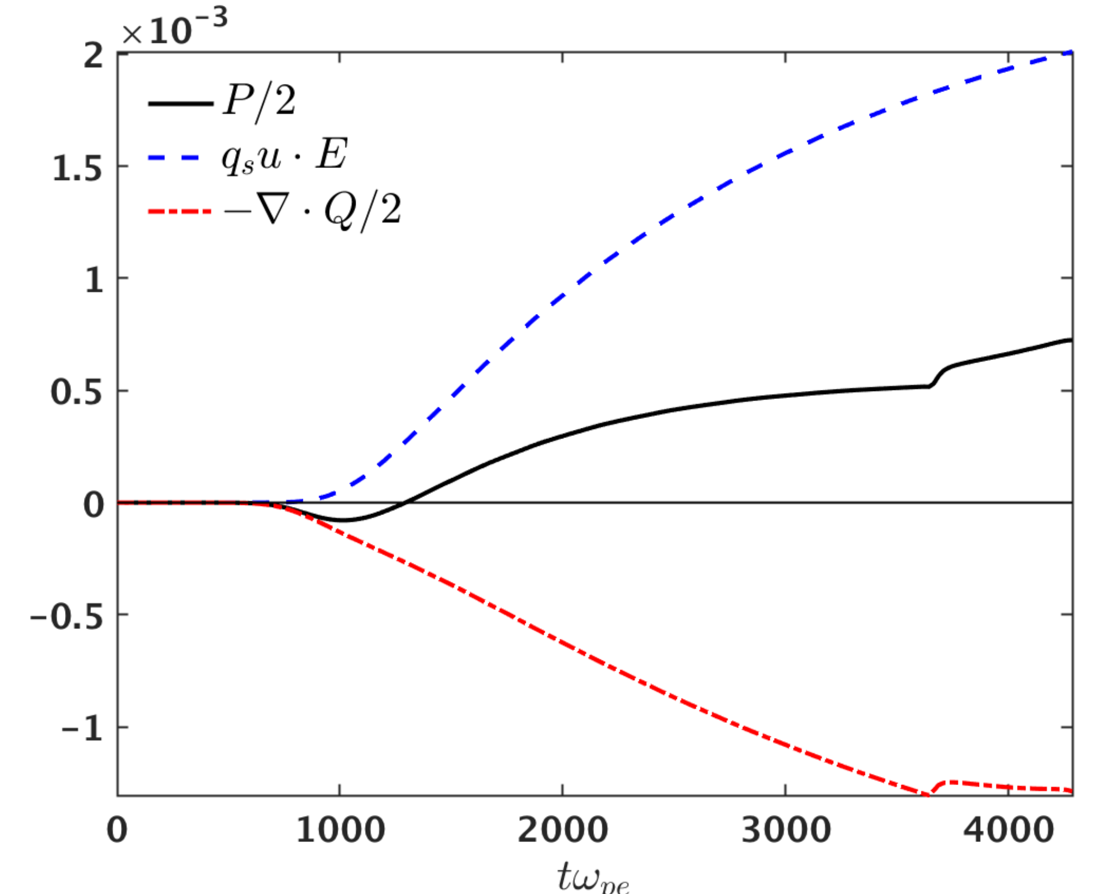
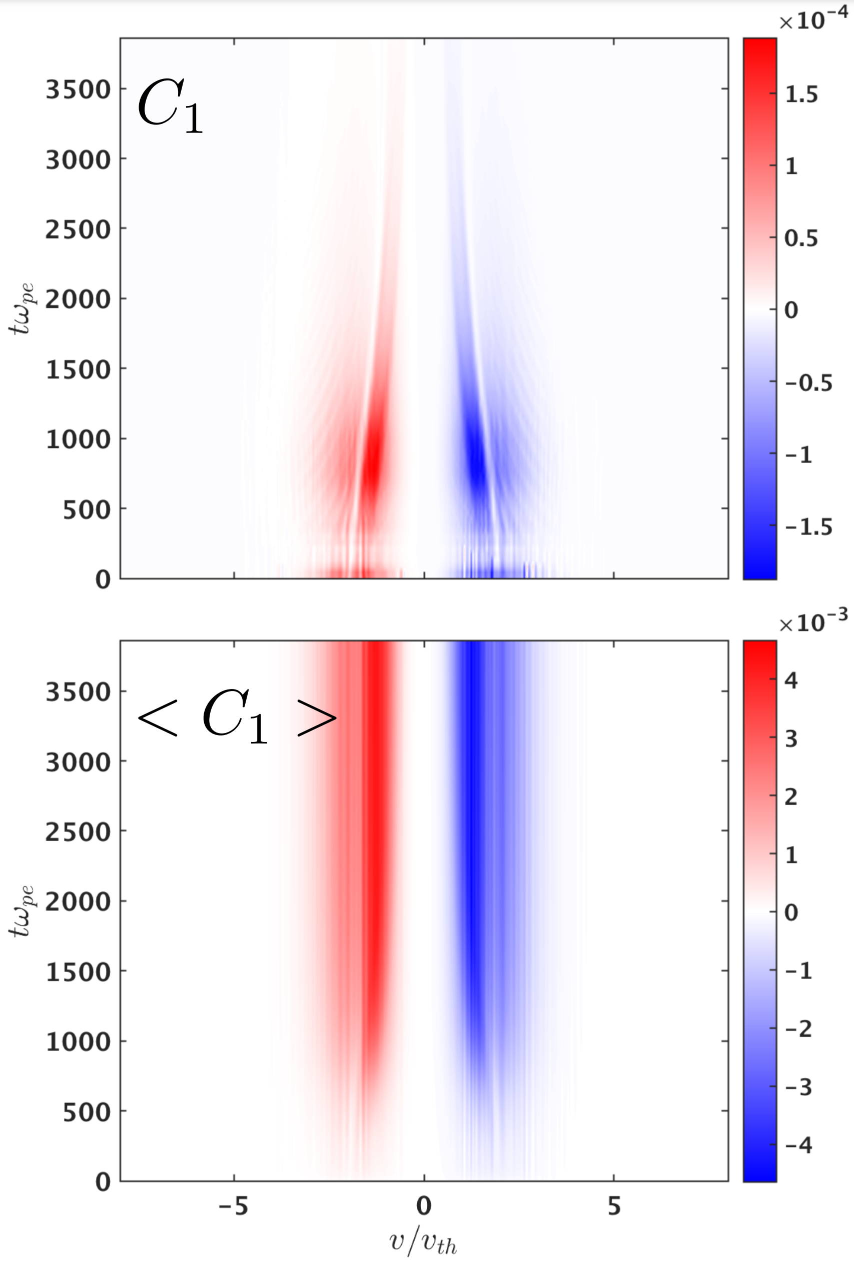
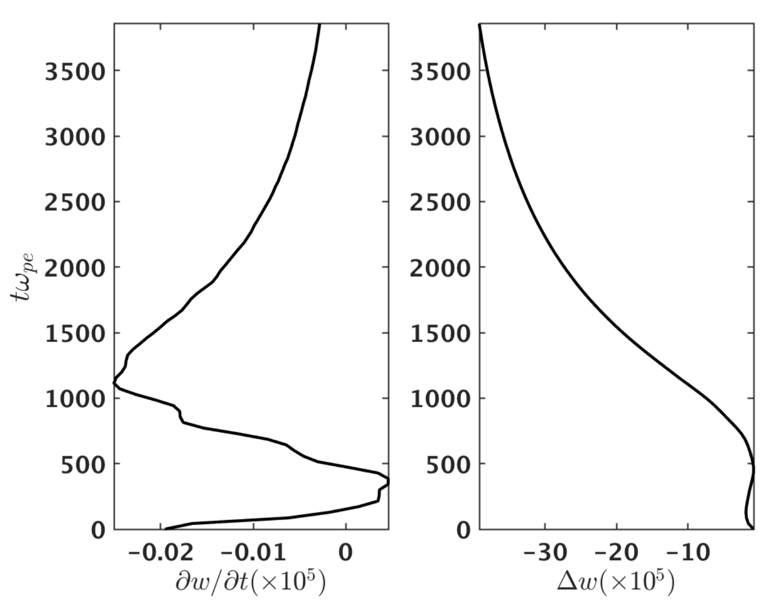
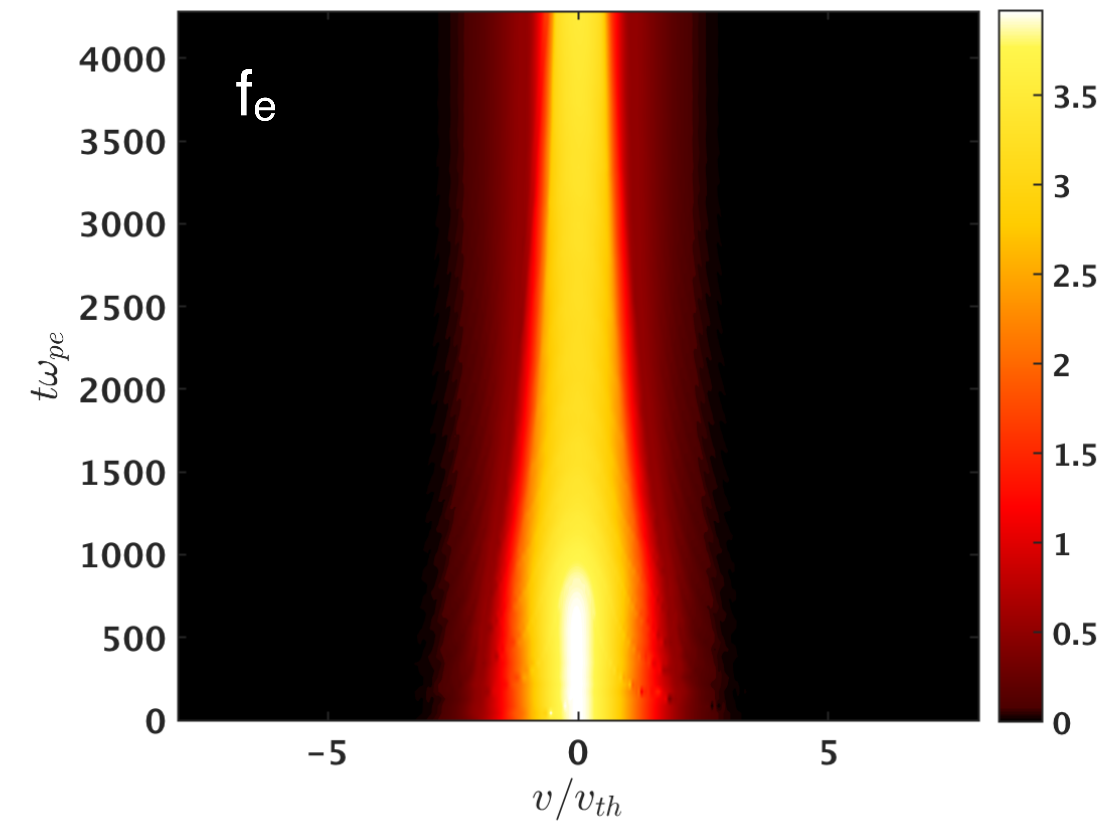


Heat flux leads to moderate cooling

Electron shock result at x=100λ_e, $\tau\omega_{pe}$=386 (Gkeyll)







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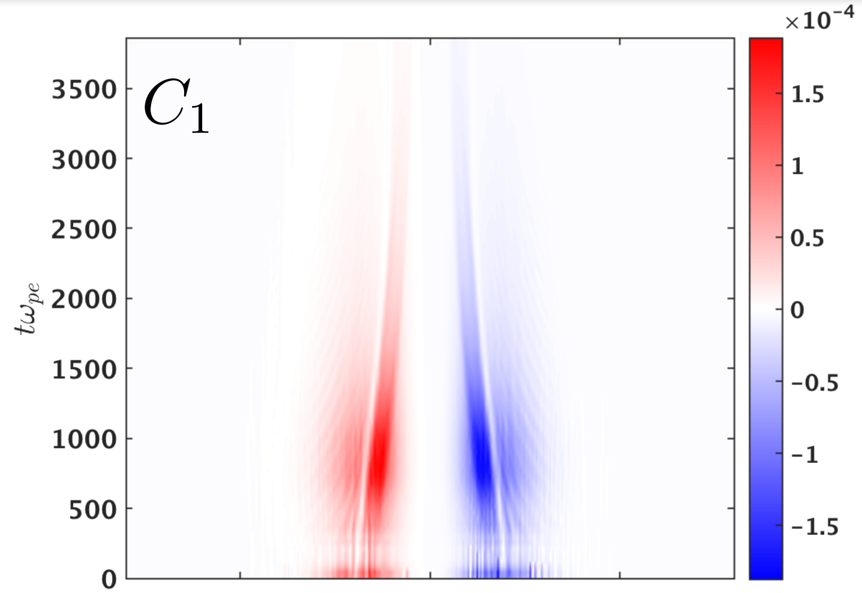

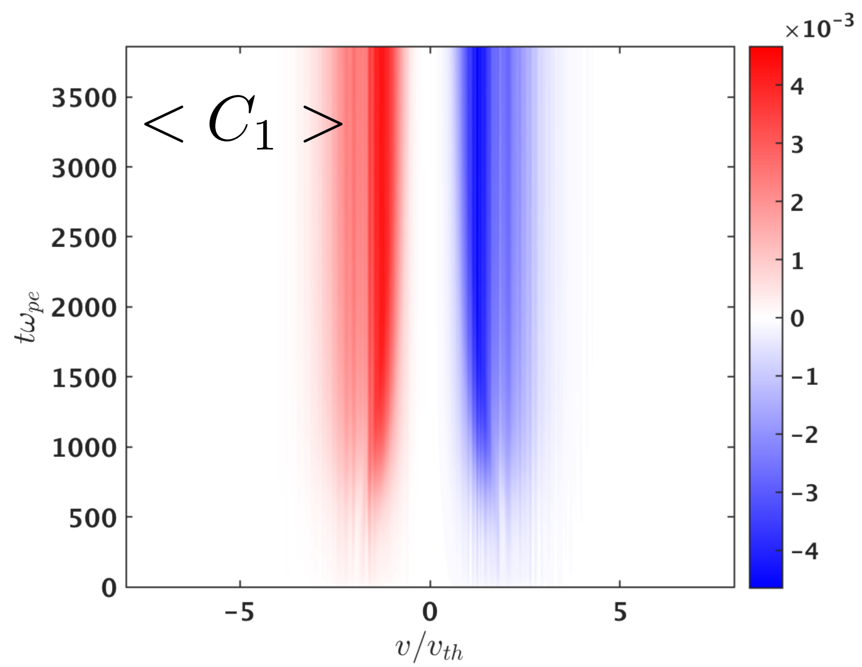

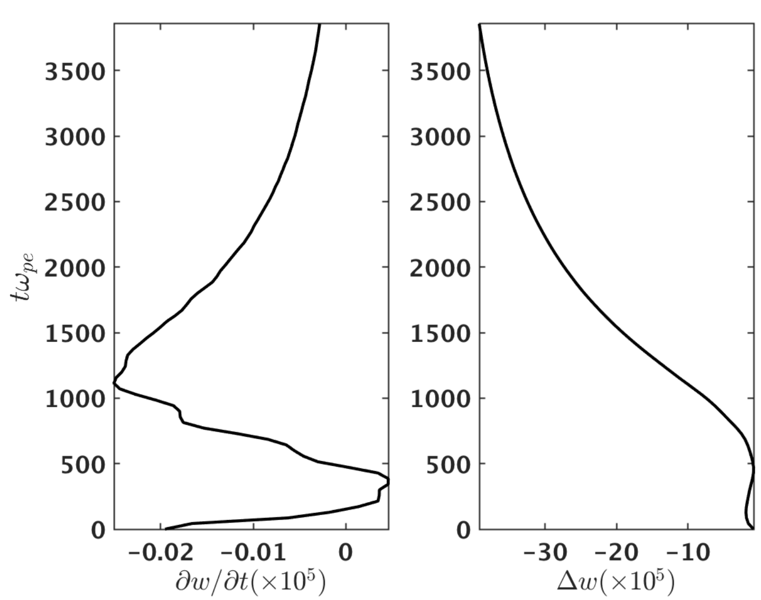

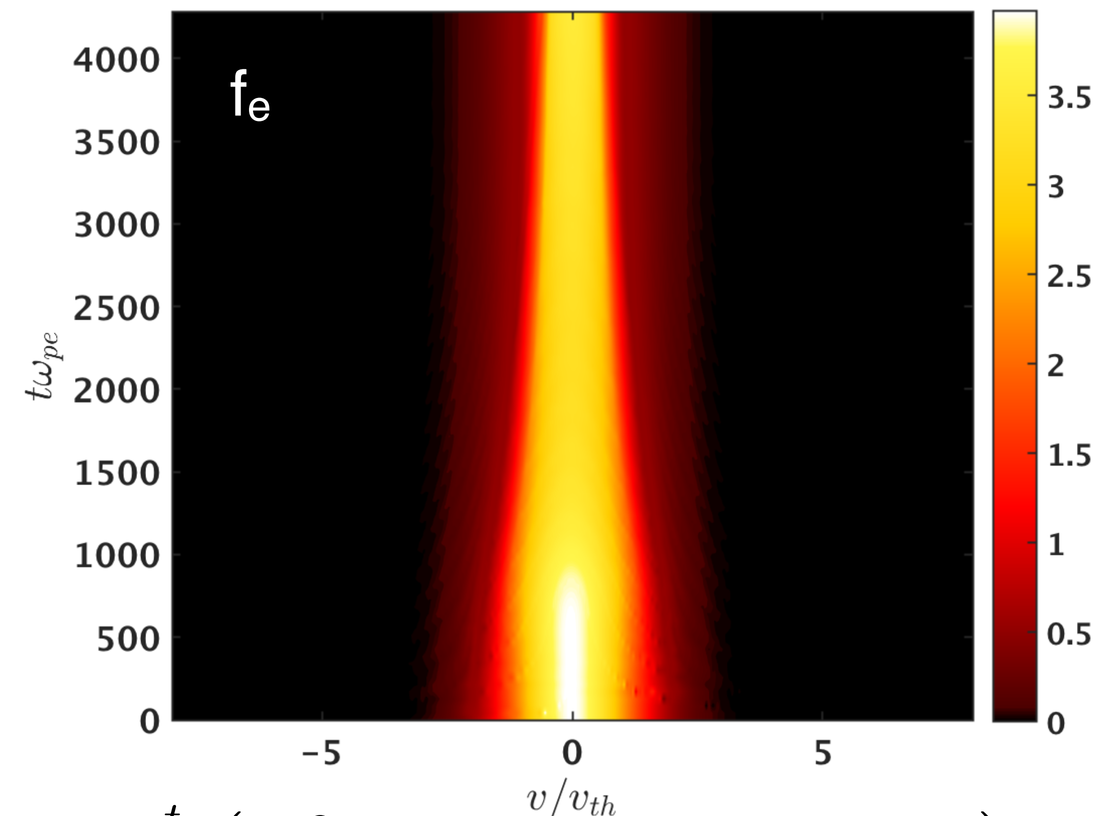

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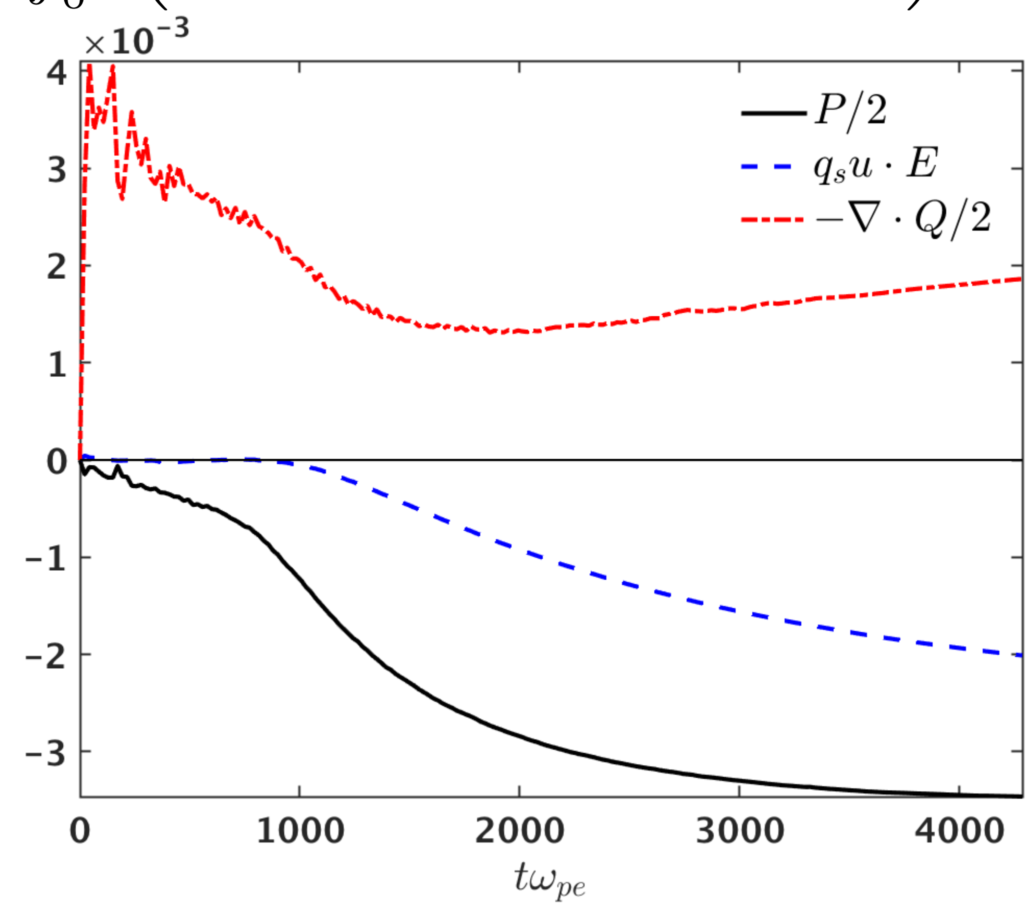
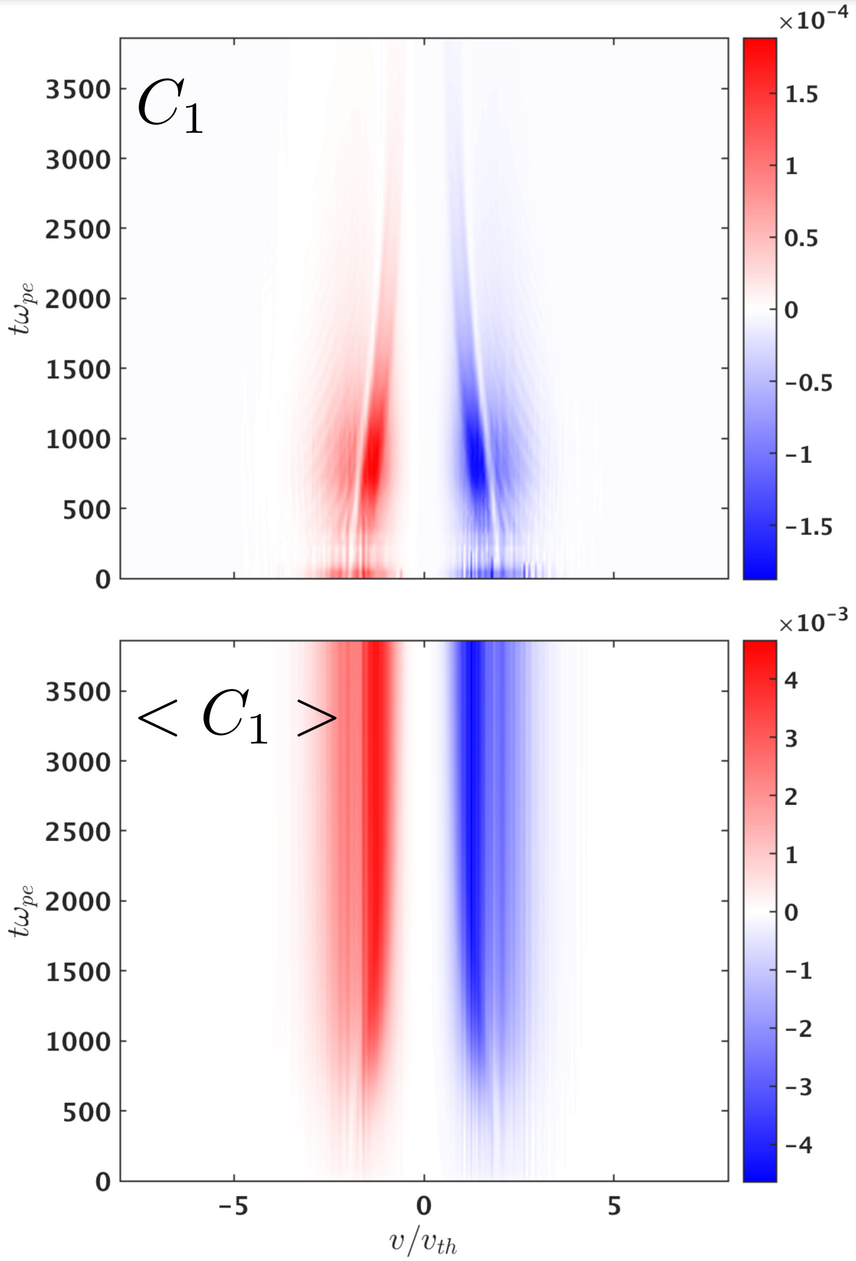
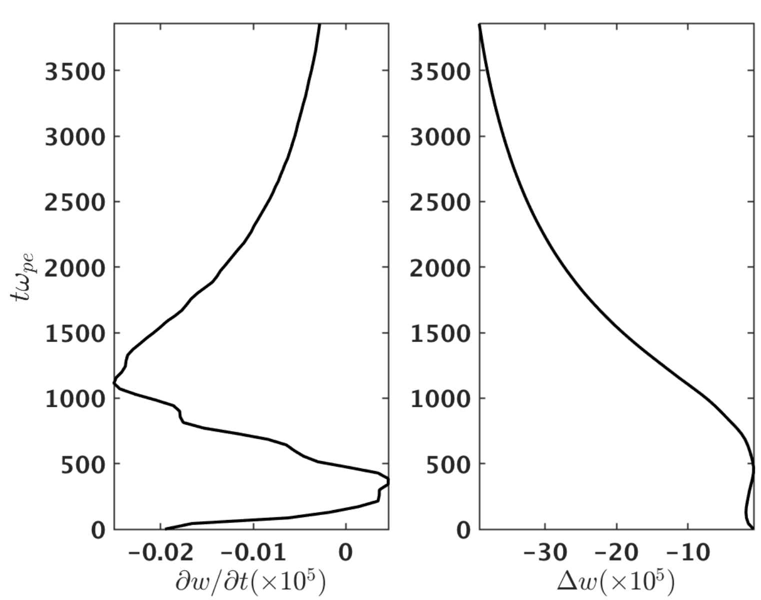


Heat flux leads to moderate cooling

Electron shock result at x=100λ_e, $\tau\omega_{pe}$=386 (Gkeyll)





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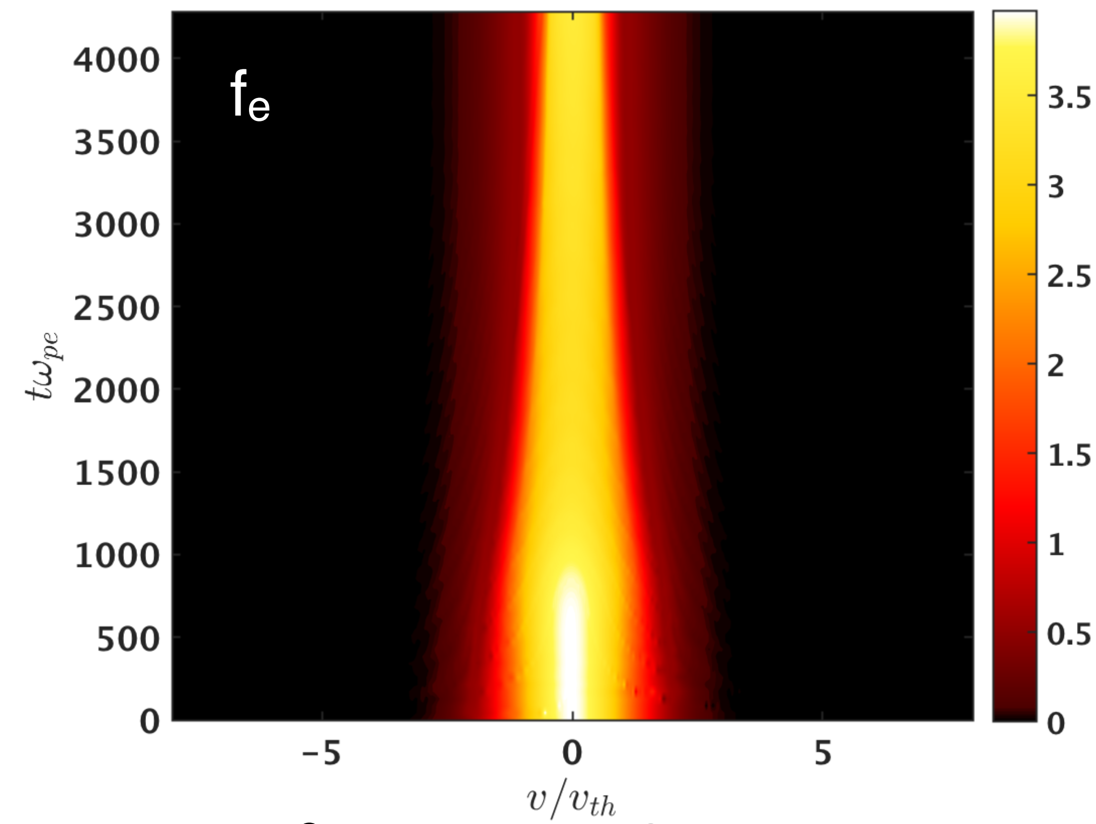

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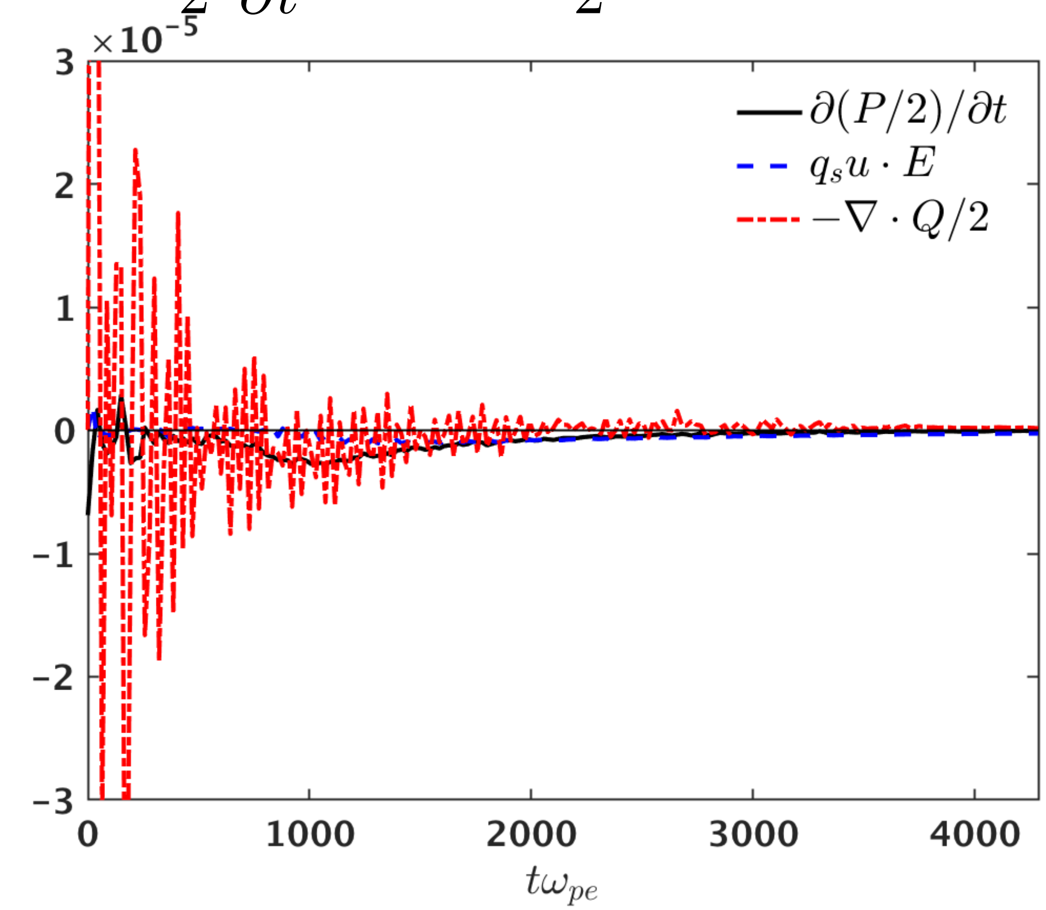
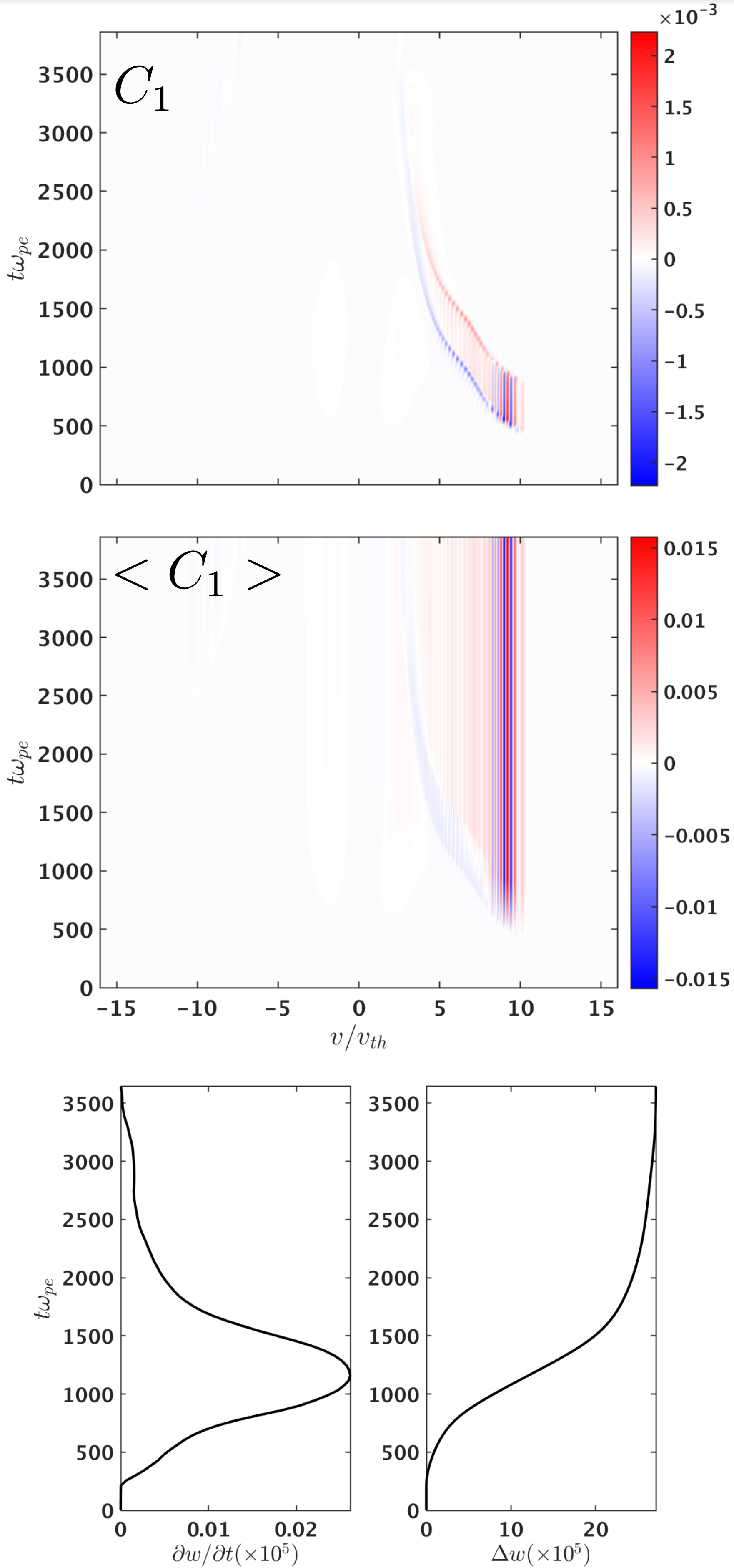
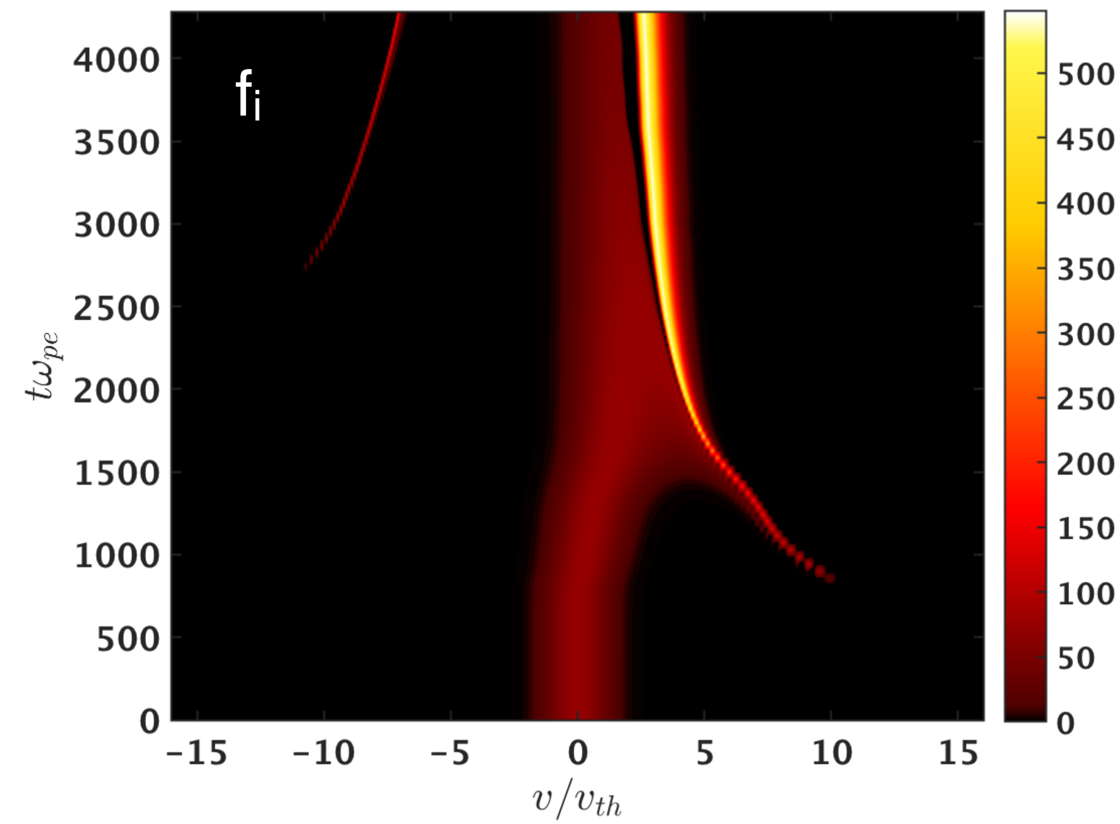


Proton shock result at $x=175\lambda_e$, $\tau\omega_{pe}=386$ (Gkeyll)





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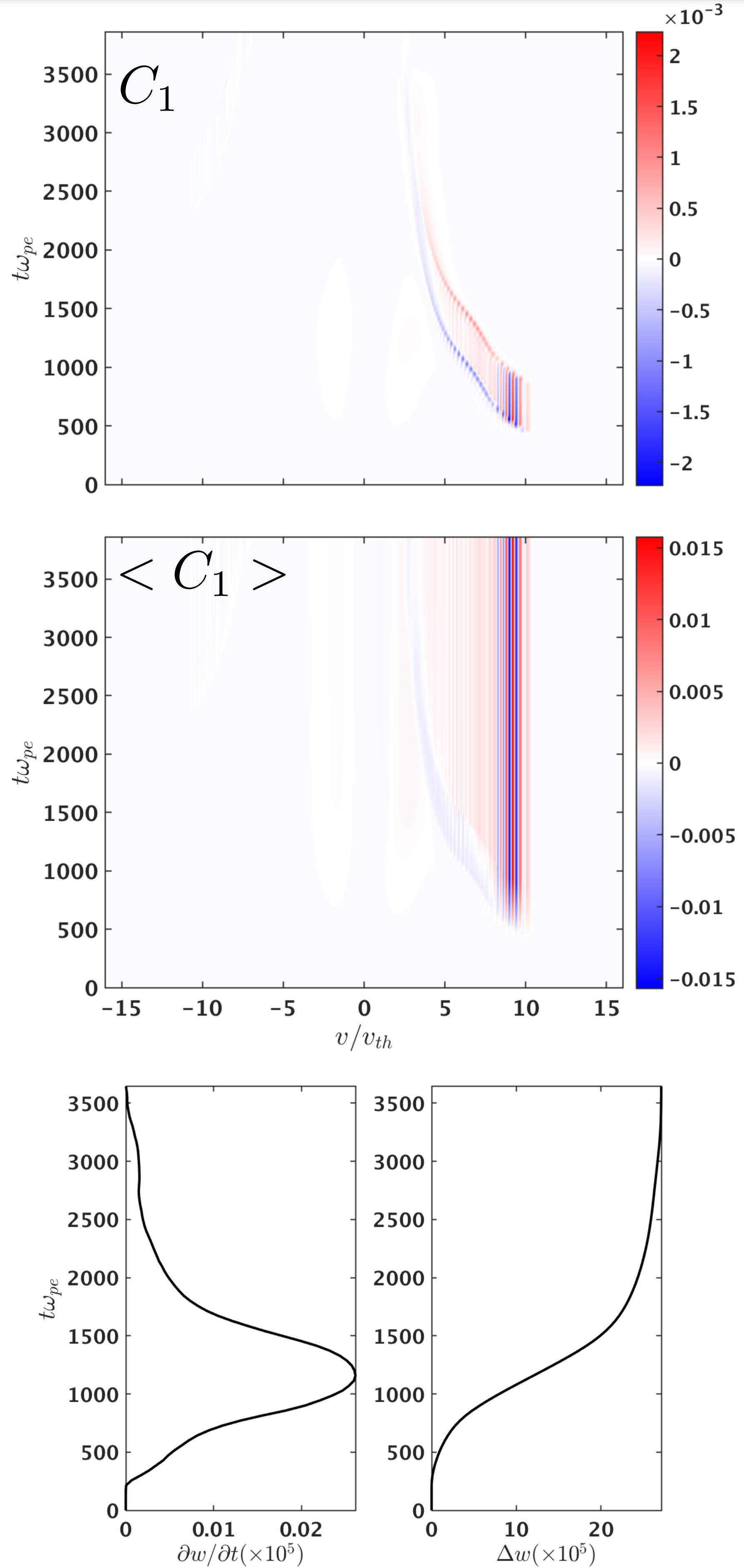
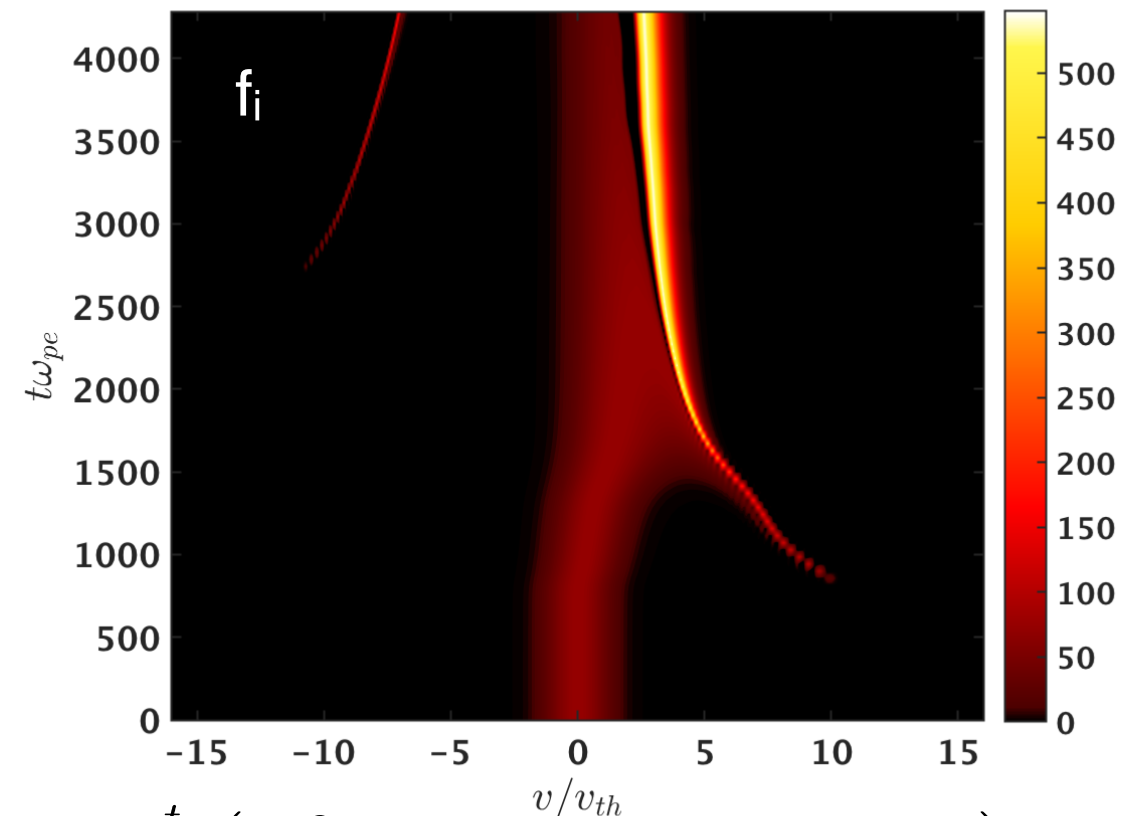
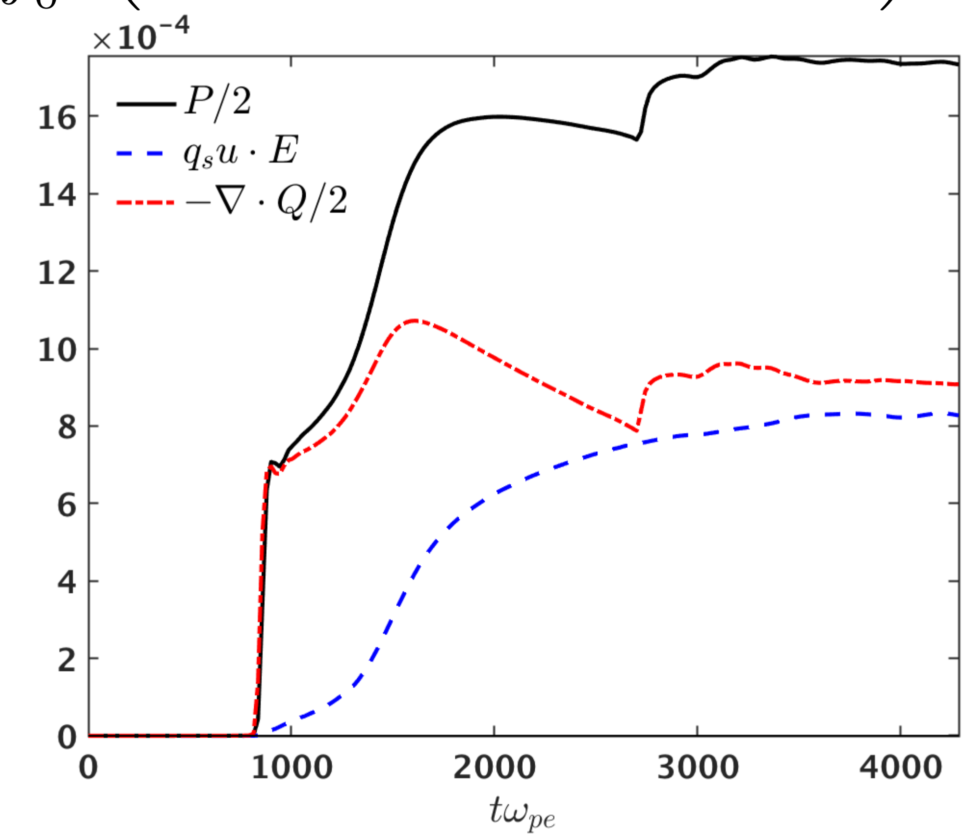


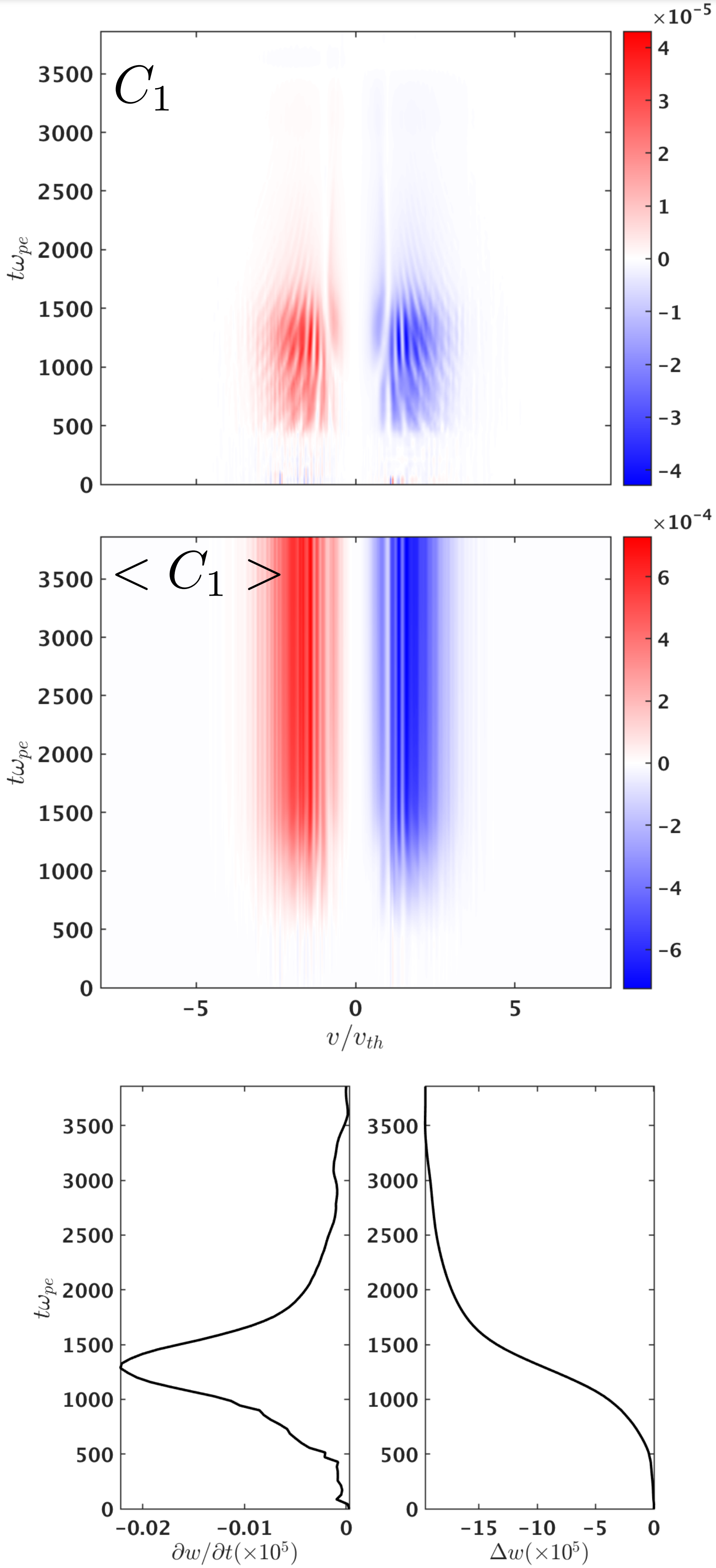


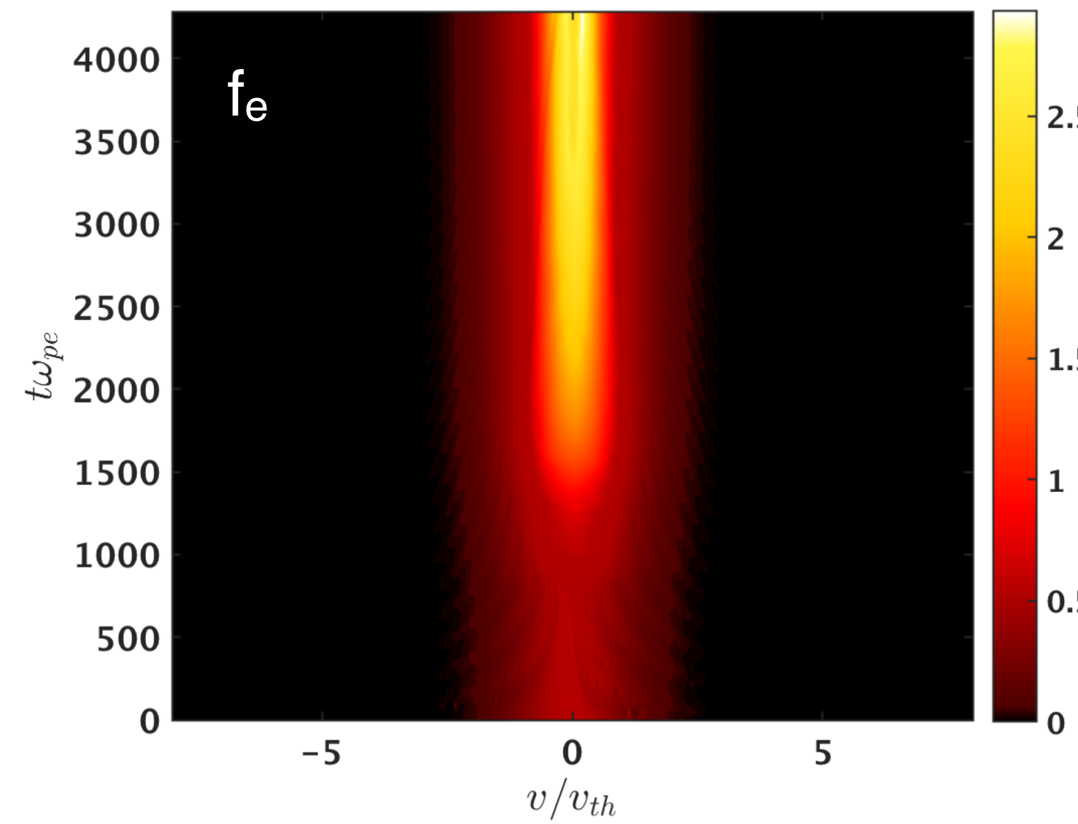
$$\int_0^t \left(\frac{1}{2}\frac{\partial P}{\partial t'} = -\nabla\cdot\frac{\mathbf{Q}}{2} + qn\mathbf{u}\cdot\mathbf{E} \right) dt'$$



Heat flux (beam) leads to significant heating

Electron shock result at x=175λ_e, $\tau\omega_{pe}$=386 (Gkeyll)





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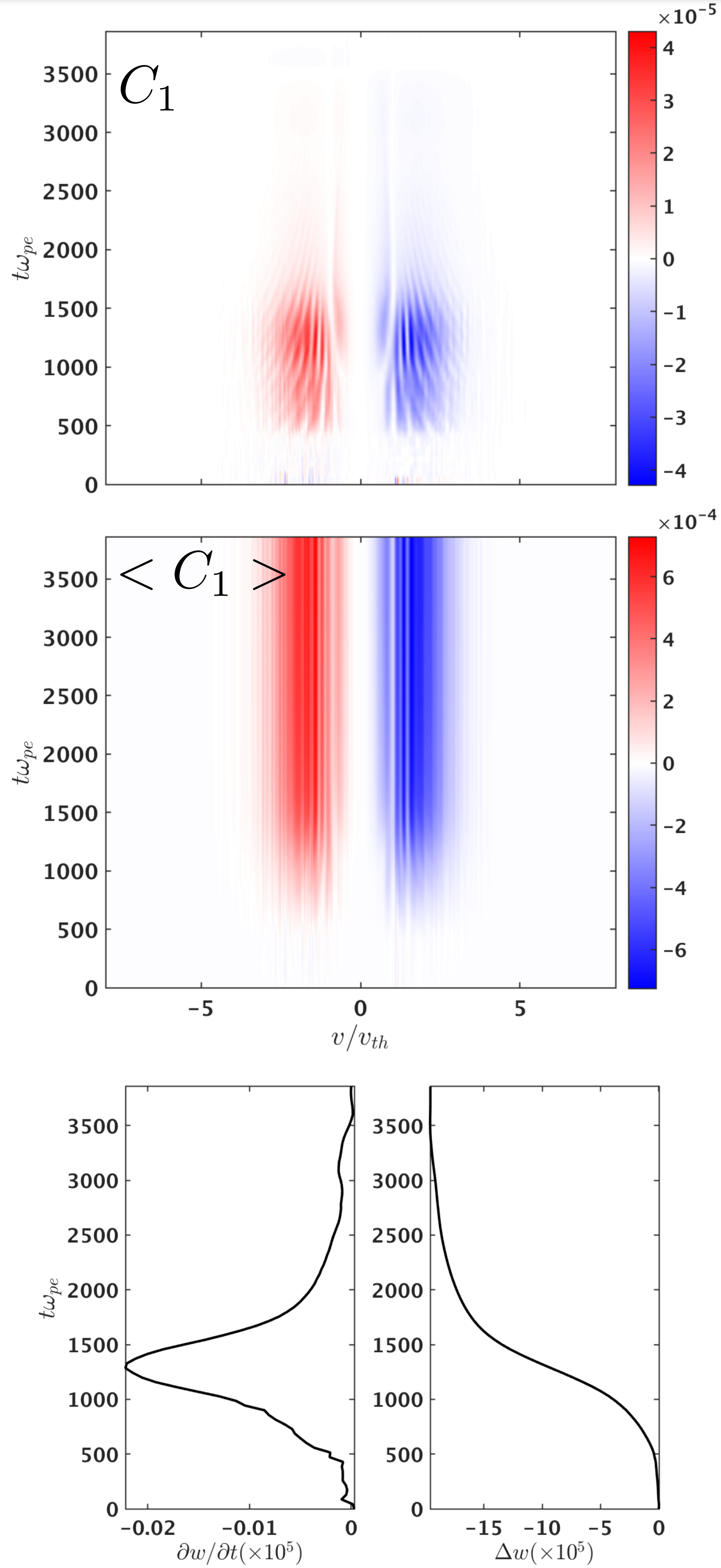

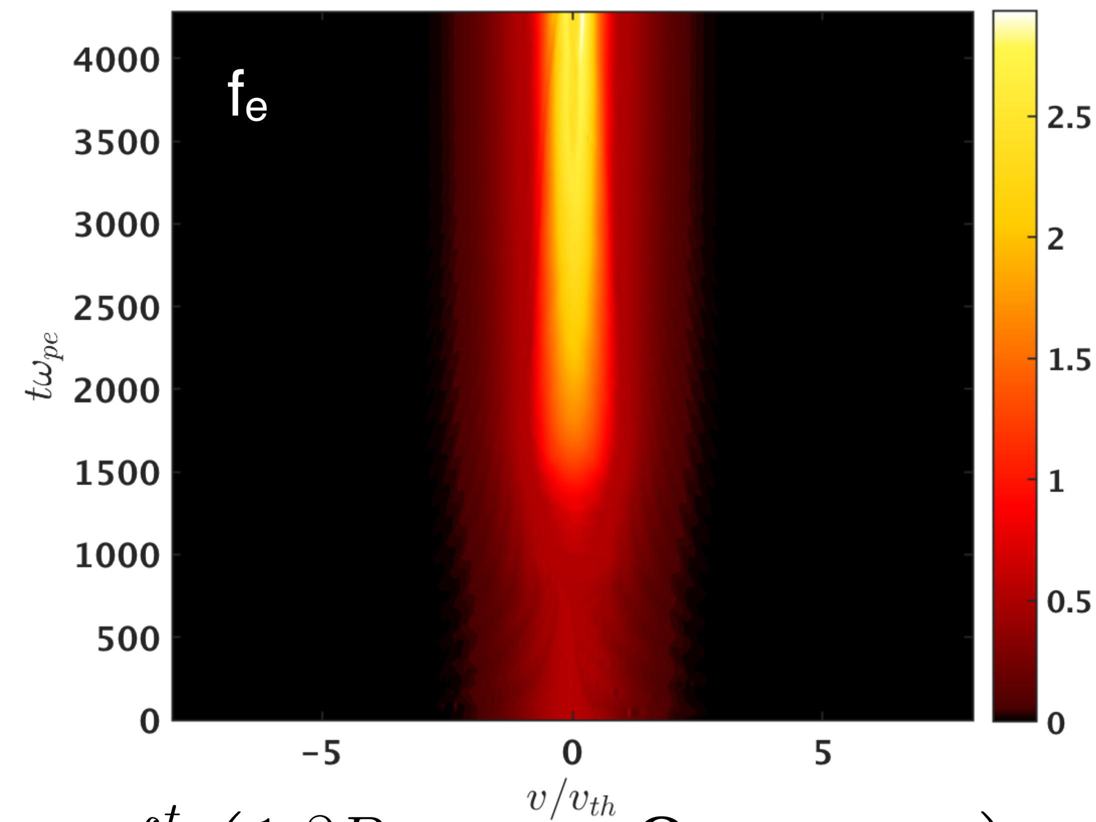

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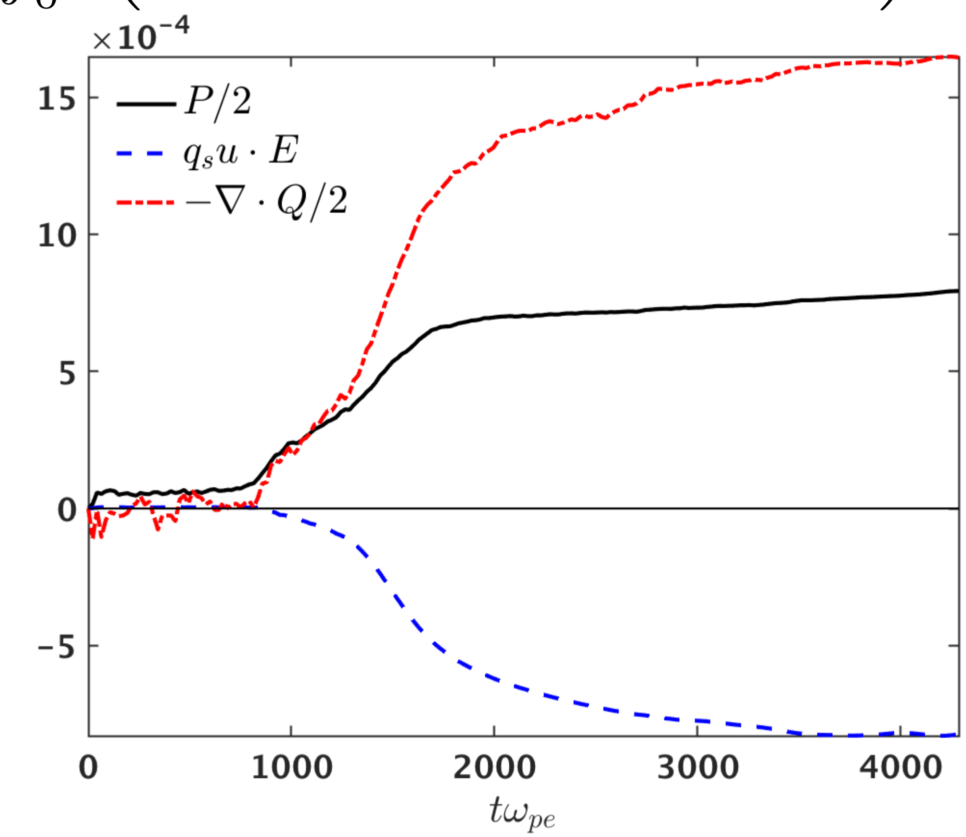


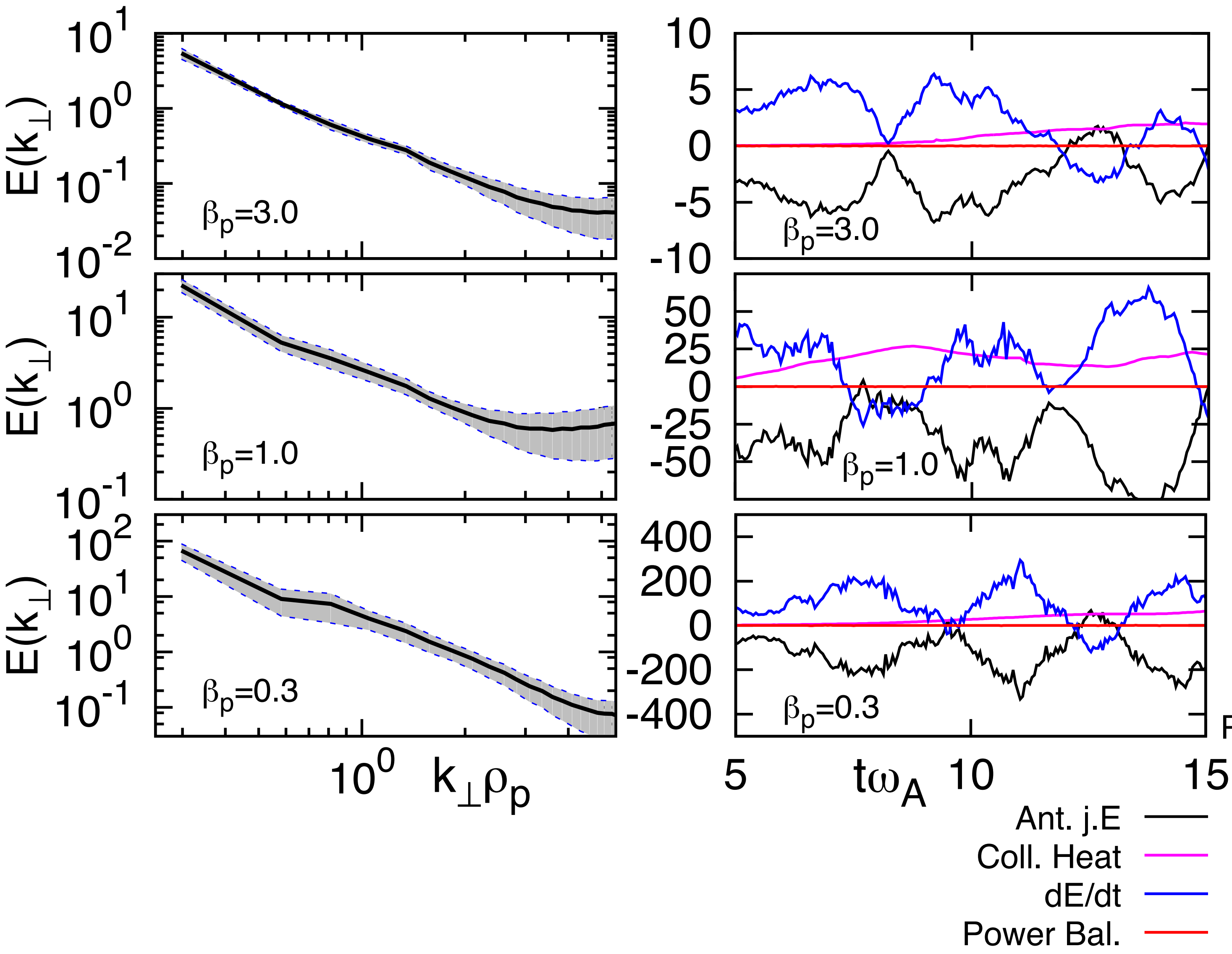
Heat flux leads to significant heating and contradicts FPC

Gyrokinetic turbulence

Gyrokinetic turbulence (AstroGK)

$$m_p/m_e = 32$$
$$k_\perp \rho_p \in [0.25, 5.5] \rightarrow L_{x,y} = 25\rho_p$$
$$(n_x, n_y, n_z, n_\lambda, n_E) = (64, 64, 32, 64, 32)$$
$$\nu_p / k_\parallel v_{tp} = 2 \times 10^{-4}$$

Driven by Langevin antenna [TenBarge et al 2014] at outer scale



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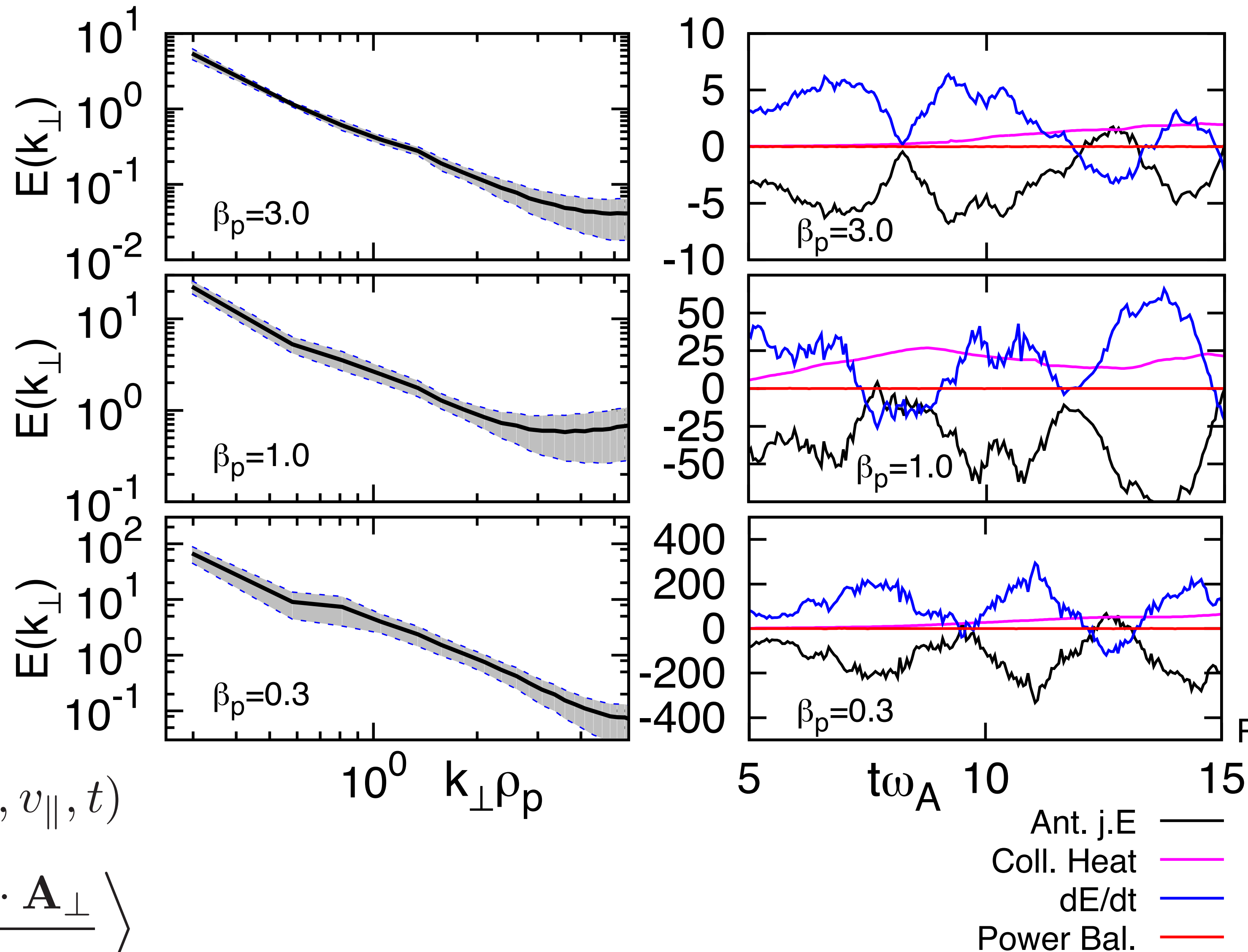
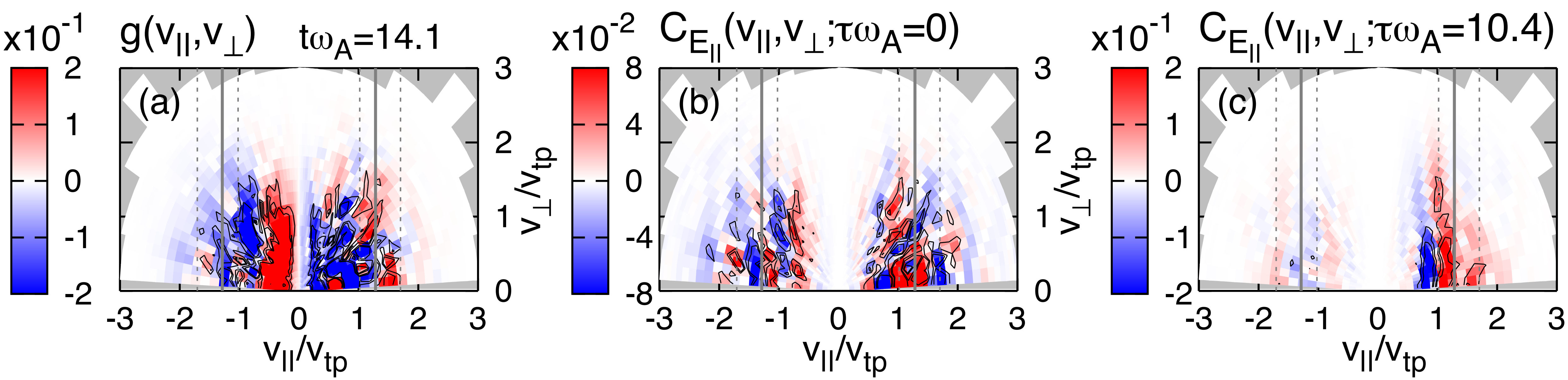
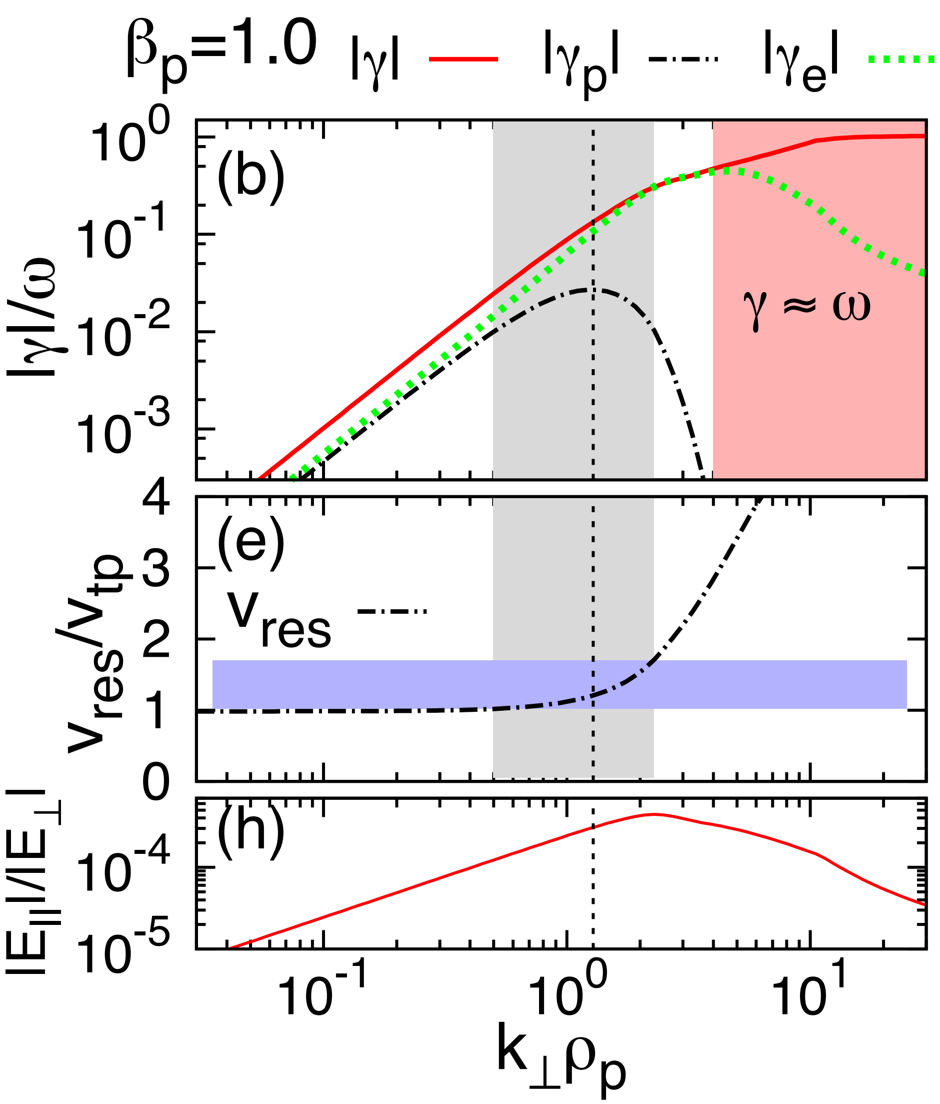


$$f_s(\mathbf{r}, \mathbf{v}, t) = F_{0s}(v)\left(1 - \frac{q_s \phi(\mathbf{r}, t)}{T_{0s}}\right) + h_s(\mathbf{R}_s, v_\perp, v_\|, t)$$

$$g_s(\mathbf{R}_s, v_\perp, v_\|) = h_s(\mathbf{R}_s, v_\perp, v_\|) - \frac{q_s F_{0s}}{T_{0s}}\left\langle \phi - \frac{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}{c} \right\rangle_{\mathbf{R}_s}$$



GK turbulence field particle correlation (AstroGK)



$$C_{E_\parallel}(\mathbf{v},t,\tau) = C\left(-q_s \frac{v_\parallel^2}{2}\frac{\partial f_s(\mathbf{r}_0,\mathbf{v},t)}{\partial v_\parallel}, E_\parallel(\mathbf{r}_0,t)\right)$$

$$C_{E_\parallel}(v_\parallel) = \int dv_\perp C_{E_\parallel}(v_\parallel, v_\perp)$$

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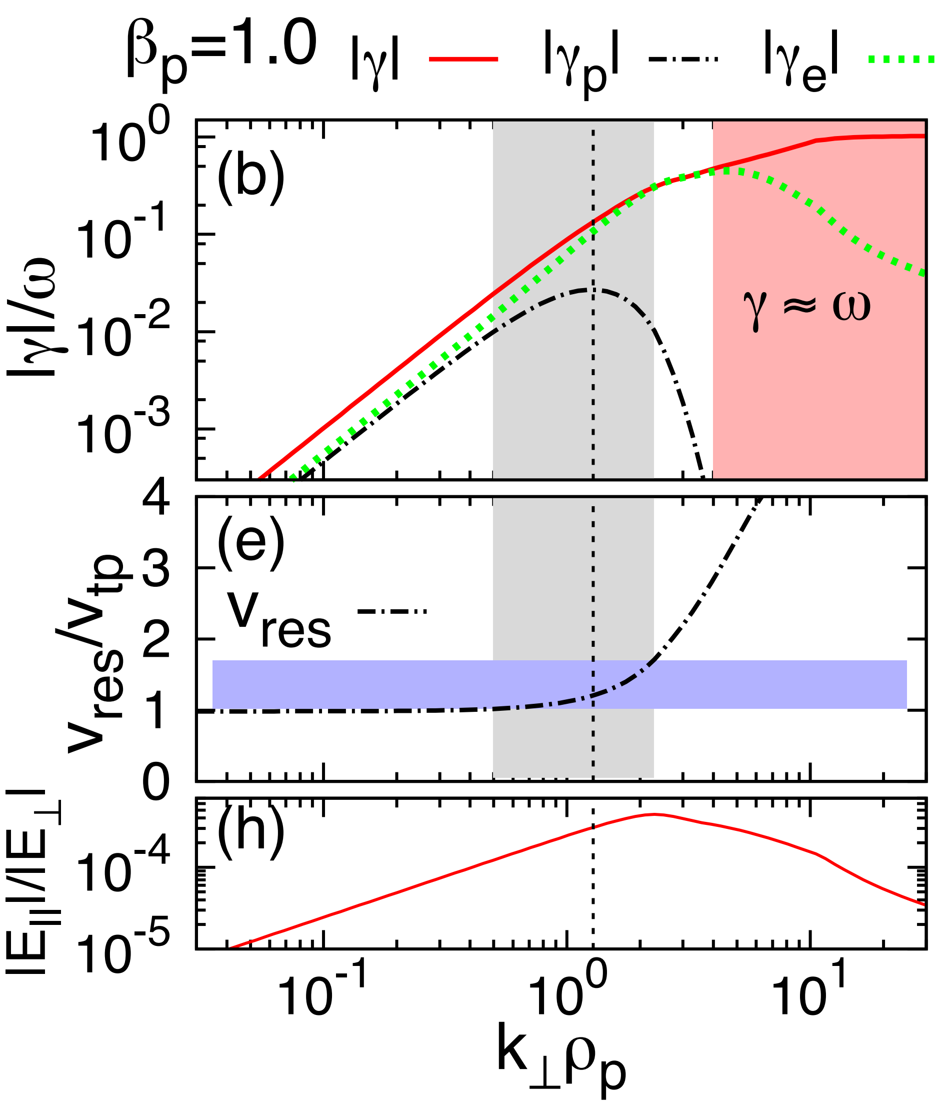

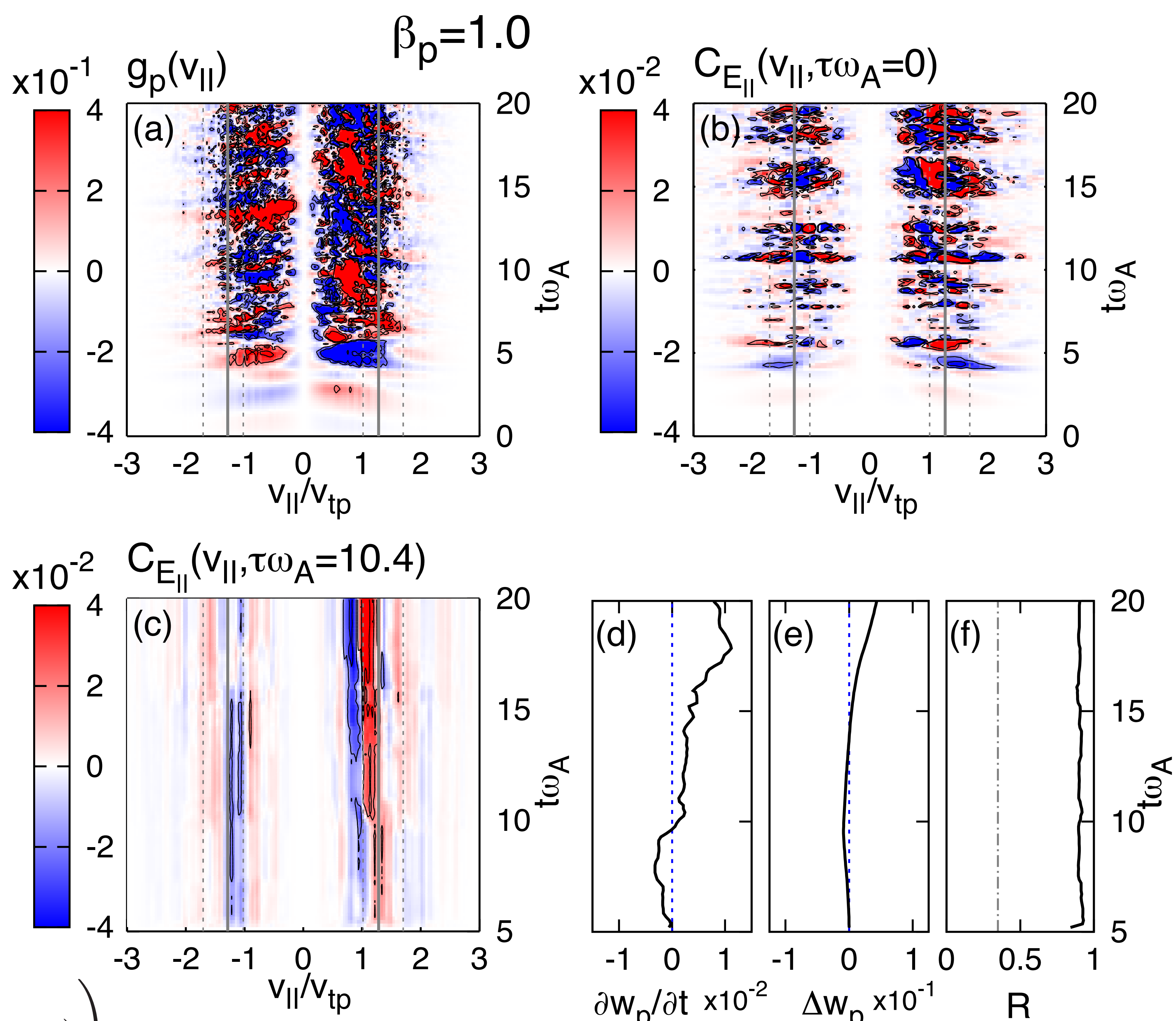

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FPC identifies dominant dissipation is consistent with KAW Landau damping

Hybrid kinetic turbulence

Hybrid turbulence field particle correlation (Pegasus)

$$\beta_p = 1$$
$$L_x \times L_y \times L_z = (20\pi)^2 \times 160\pi\rho_p$$
$$(n_x, n_y, n_z, n_{ppc}) = (200, 200, 1600, 512)$$

$$\beta_p = 0.3$$
$$L_x \times L_y \times L_z = (20\pi)^2 \times 120\pi\rho_p$$
$$(n_x, n_y, n_z, n_{ppc}) = (200, 200, 1200, 512)$$

Driven at largest scales in domain with a finite time correlated force satisfying ▽.F = 0

$$C_{\parallel}(v, t, \tau) = C\left(-q_s \frac{v^2}{2} \frac{\partial f(x_0, v, t)}{\partial v_{\parallel}}, E_{\parallel}(x_0, t)\right)$$

$$C_{\perp}(v, t, \tau) = C\left(-q_s \frac{v^2}{2} \frac{\partial f(x_0, v, t)}{\partial \mathbf{v}_{\perp}}, \mathbf{E}_{\perp}(x_0, t)\right)$$

From Arzamasskiy, Kunz, Chandran, Quataert (2018)

Hybrid turbulence field particle correlation (Pegasus)

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Driven at largest scales in domain with a finite time correlated force satisfying ▽.F = 0

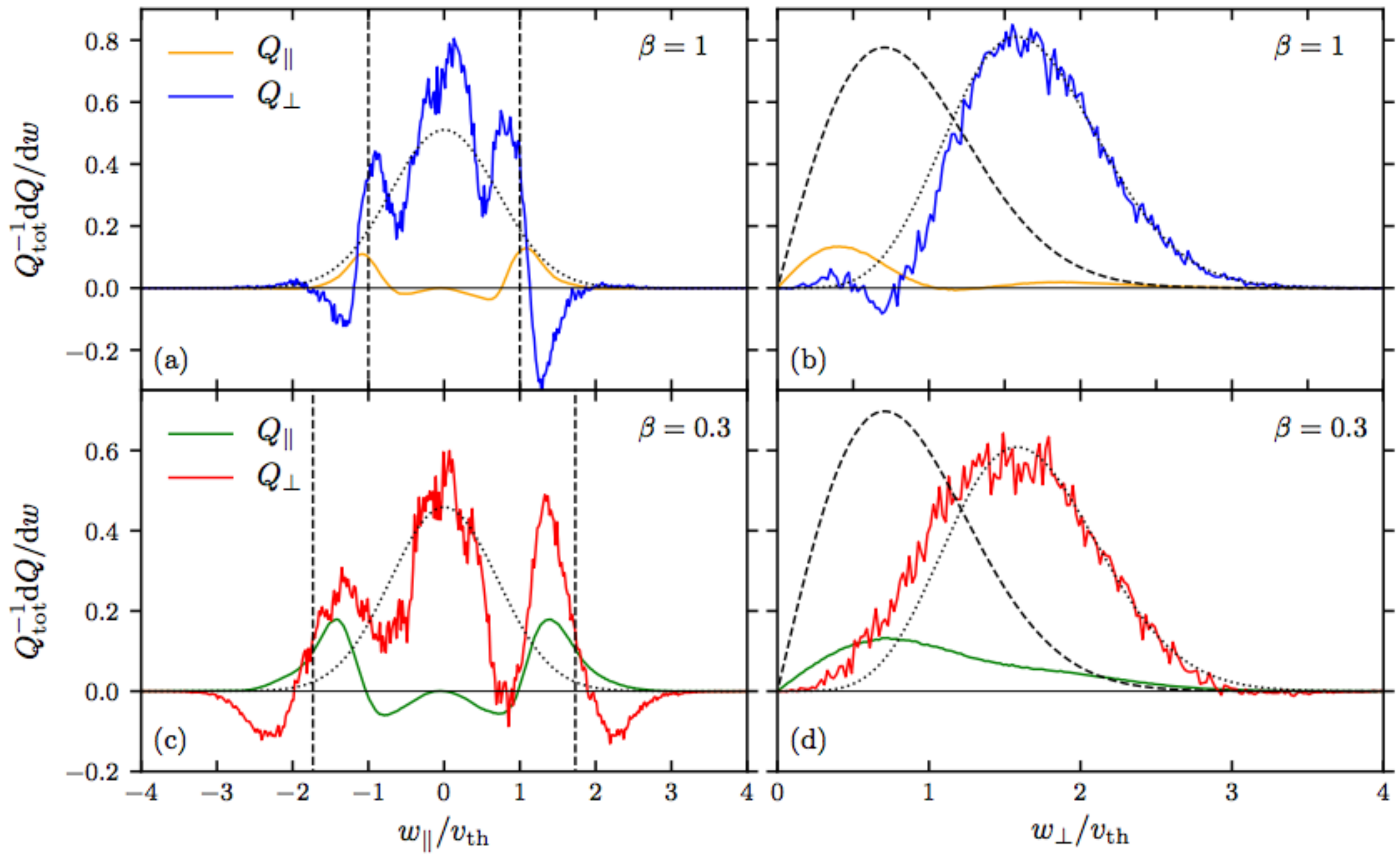
$$Q_{\|} = q\mathbf{v}_{\|}\mathbf{E}_{\|}$$

$$Q_{\perp} = q\mathbf{v}_{\perp} \cdot \mathbf{E}_{\perp}$$

From Arzamasskiy, Kunz, Chandran, Quataert (2018)

Hybrid turbulence field particle correlation (Pegasus)

- Signature of Landau damping of KAWs visible, but …
- Dominantly perpendicular heating due to high frequency stochastic heating



$$Q_{\parallel} = q\mathbf{v}_{\parallel}\mathbf{E}_{\parallel}$$

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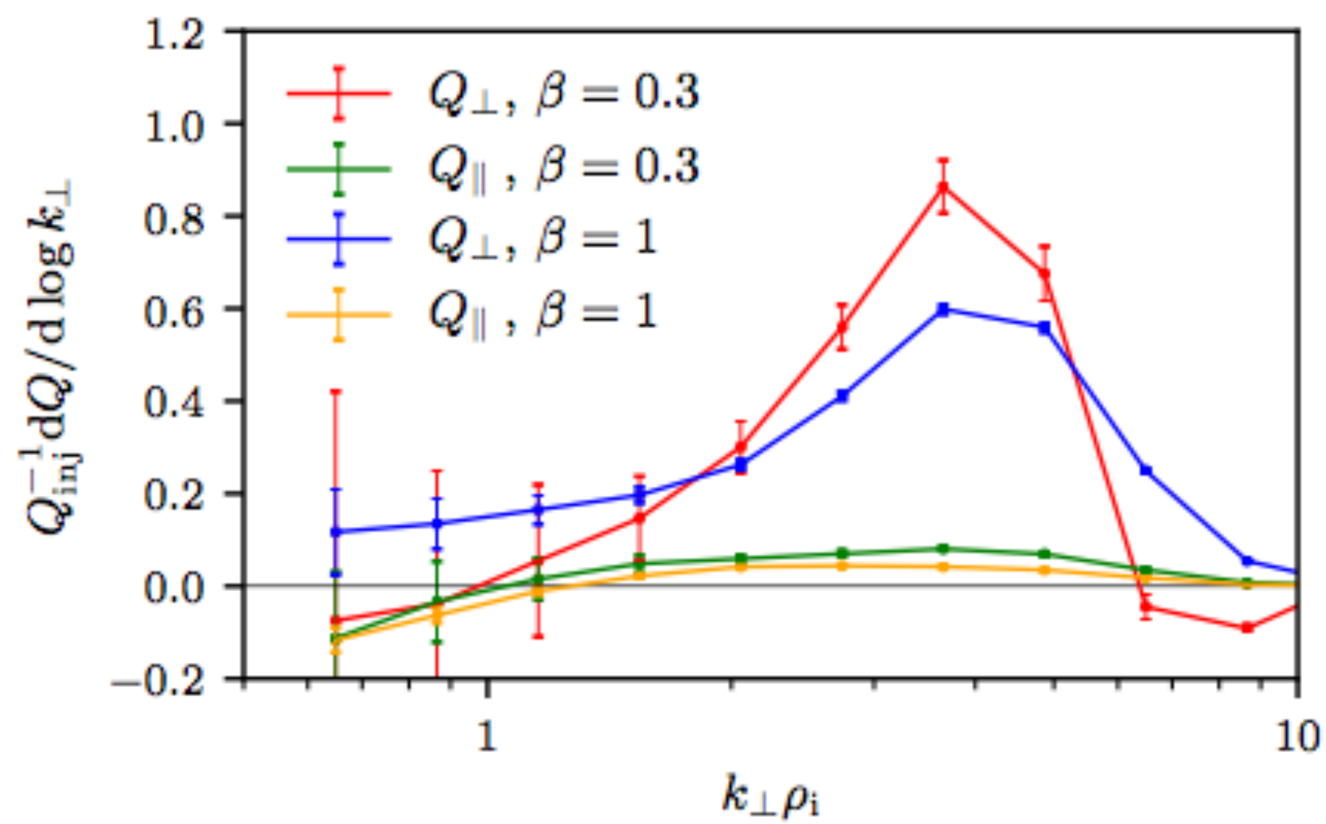

From Arzamasskiy, Kunz, Chandran, Quataert (2018)

Discussion of strengths and weaknesses FPC

- Provides a clear signature of energy exchange for a variety of processes, including resonant wave-particle interactions
- Signatures of other processes are yet to be identified but likely exist
- Provides a local, single point, measure of the energy exchange
  - Great for spacecraft data
  - In some cases, it may neglect the dominant source of energy transfer
- The connection to total energy exchange is simply $J_s.E$
- The FPC is connected to heating but indirectly through phase mixing and collisionality

Pi-D diagnostic

Introduction of the PiD diagnostic [Yang et al (2017)]

Beginning with the standard extended MHD equations

$$\partial_t \rho_\alpha + \nabla \cdot (\rho_\alpha \boldsymbol{u}_\alpha) = 0,$$

$$\partial_t (\rho_\alpha \boldsymbol{u}_\alpha) + \nabla \cdot (\rho_\alpha \boldsymbol{u}_\alpha \boldsymbol{u}_\alpha) = -\nabla \cdot \boldsymbol{P}_\alpha + n_\alpha q_\alpha (\boldsymbol{E} + \boldsymbol{u}_\alpha / c \times \boldsymbol{B}),$$

$$\partial_t \mathcal{E}_\alpha + \nabla \cdot (\mathcal{E}_\alpha \boldsymbol{u}_\alpha) = -\nabla \cdot (\boldsymbol{P}_\alpha \cdot \boldsymbol{u}_\alpha) - \nabla \cdot \mathbf{h}_\alpha + n_\alpha q_\alpha \boldsymbol{E} \cdot \boldsymbol{u}_\alpha.$$

Re-arrange to construct the:

Flow energy: $$\partial_t \mathcal{E}^f_\alpha + \nabla \cdot (\mathcal{E}^f_\alpha \boldsymbol{u}_\alpha) = -\nabla \cdot (\boldsymbol{P}_\alpha \cdot \boldsymbol{u}_\alpha) + (\boldsymbol{P}_\alpha \cdot \nabla) \cdot \boldsymbol{u}_\alpha + n_\alpha q_\alpha \boldsymbol{E} \cdot \boldsymbol{u}_\alpha.$$

Rest-frame (thermal) energy: $$\partial_t \mathcal{E}^{th}_\alpha + \nabla \cdot (\mathcal{E}^{th}_\alpha \boldsymbol{u}_\alpha) = -(\boldsymbol{P}_\alpha \cdot \nabla) \cdot \boldsymbol{u}_\alpha - \nabla \cdot \mathbf{h}_\alpha$$

Electromagnetic energy: $$\partial_t \mathcal{E}^m + \frac{c}{4\pi} \nabla \cdot (\boldsymbol{E} \times \boldsymbol{B}) = -\boldsymbol{E} \cdot \boldsymbol{j}$$

Introduction of the PiD diagnostic [Yang et al (2017)]

$$\partial_t \mathcal{E}^f_\alpha + \nabla \cdot (\mathcal{E}^f_\alpha \boldsymbol{u}_\alpha) = -\nabla \cdot (\boldsymbol{P}_\alpha \cdot \boldsymbol{u}_\alpha) + (\boldsymbol{P}_\alpha \cdot \nabla) \cdot \boldsymbol{u}_\alpha + n_\alpha q_\alpha \boldsymbol{E} \cdot \boldsymbol{u}_\alpha.$$

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$$\partial_t \mathcal{E}^m + \frac{c}{4\pi} \nabla \cdot (\boldsymbol{E} \times \boldsymbol{B}) = -\boldsymbol{E} \cdot \boldsymbol{j}$$

$\xrightarrow{\text{Integrating over space}}$

$$\partial_t \langle \mathcal{E}^f_\alpha \rangle = \langle (\boldsymbol{P}_\alpha \cdot \nabla) \cdot \boldsymbol{u}_\alpha \rangle + \langle n_\alpha q_\alpha \boldsymbol{E} \cdot \boldsymbol{u}_\alpha \rangle,$$

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Integrating over space $\longrightarrow$

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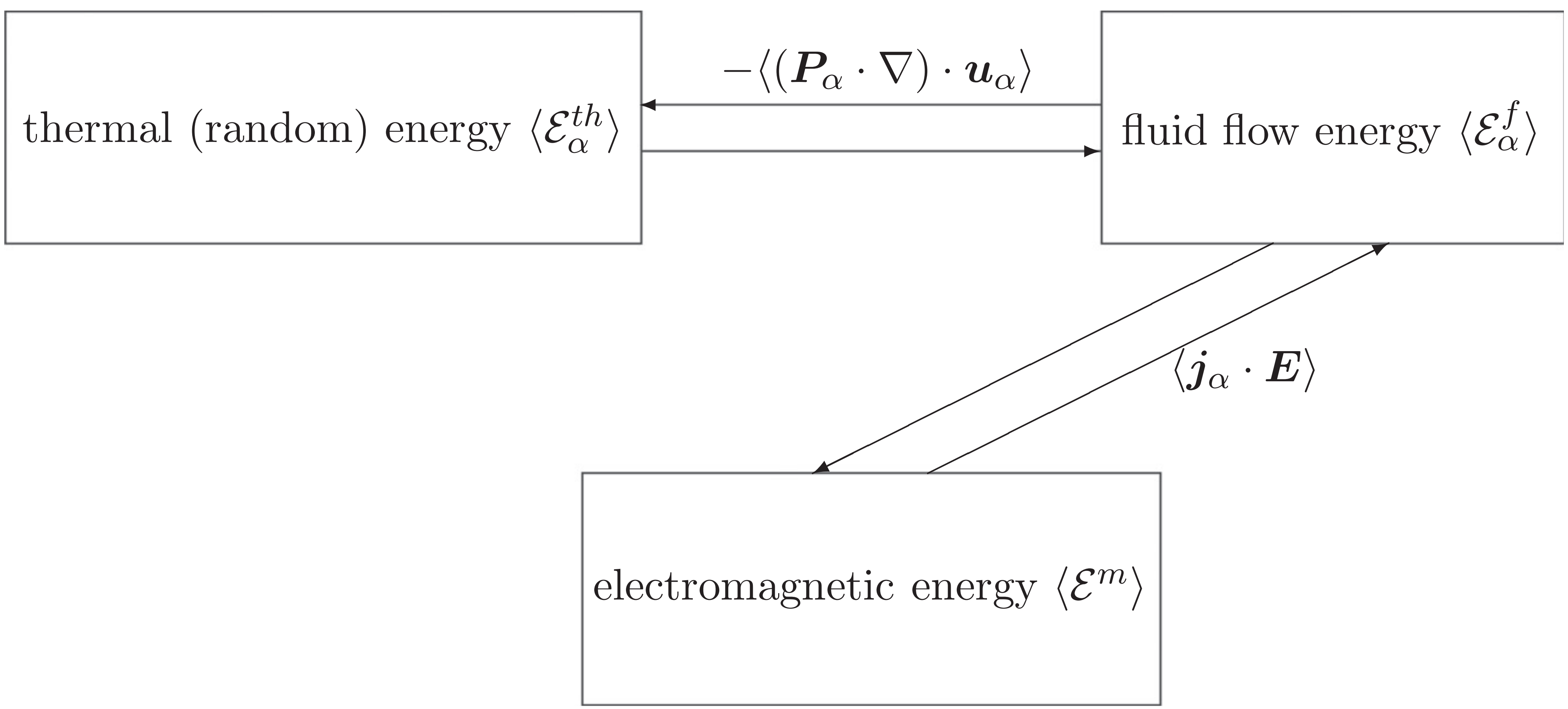
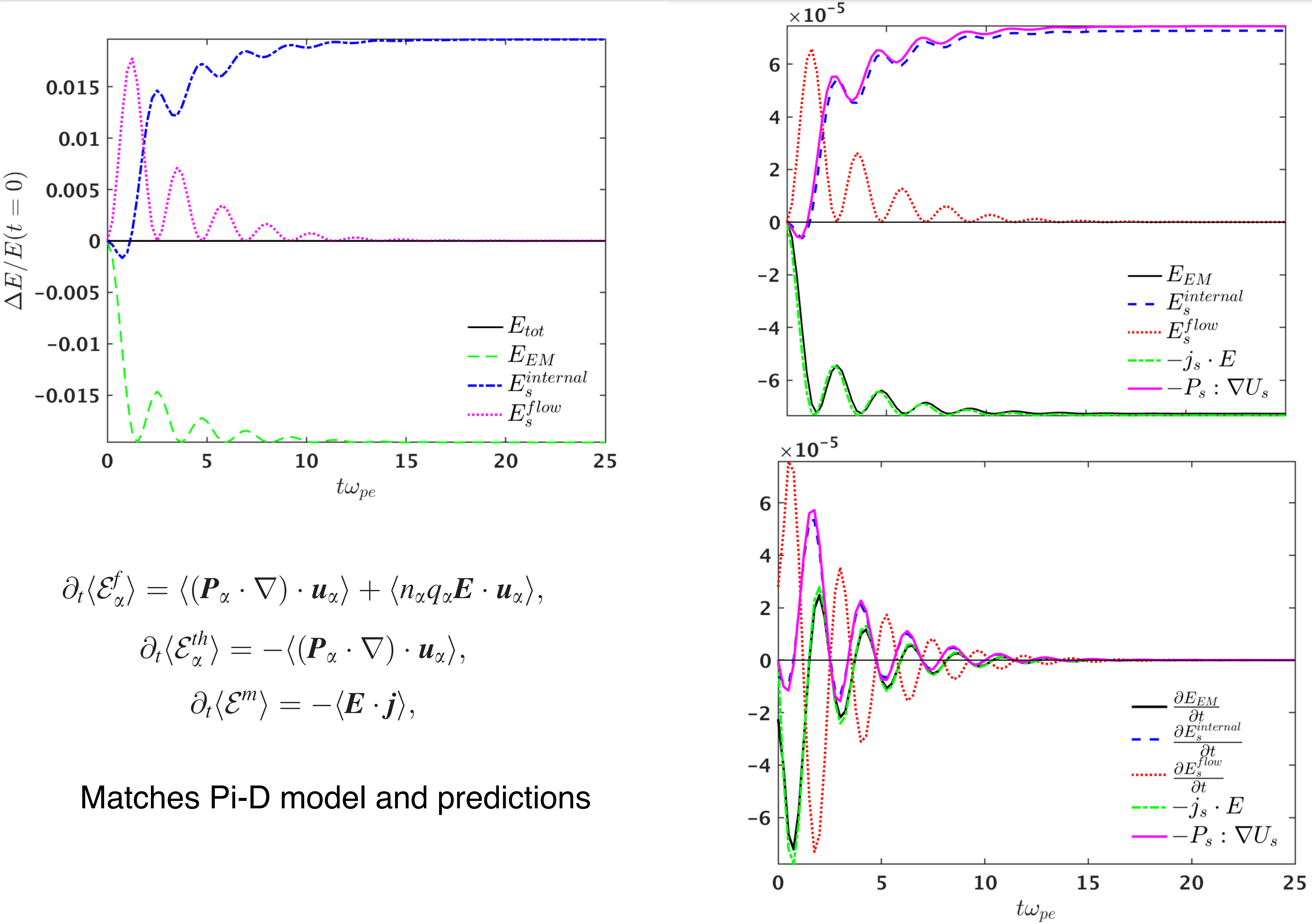


Revisiting the Langmuir wave

Langmuir wave result



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Matches Pi-D model and predictions

Revisiting the Sod shock

Sod shock spatially integrated electrons

$$\partial_t \langle \mathcal{E}^f_\alpha \rangle = \langle (\boldsymbol{P}_\alpha \cdot \nabla) \cdot \boldsymbol{u}_\alpha \rangle + \langle n_\alpha q_\alpha \boldsymbol{E} \cdot \boldsymbol{u}_\alpha \rangle,$$

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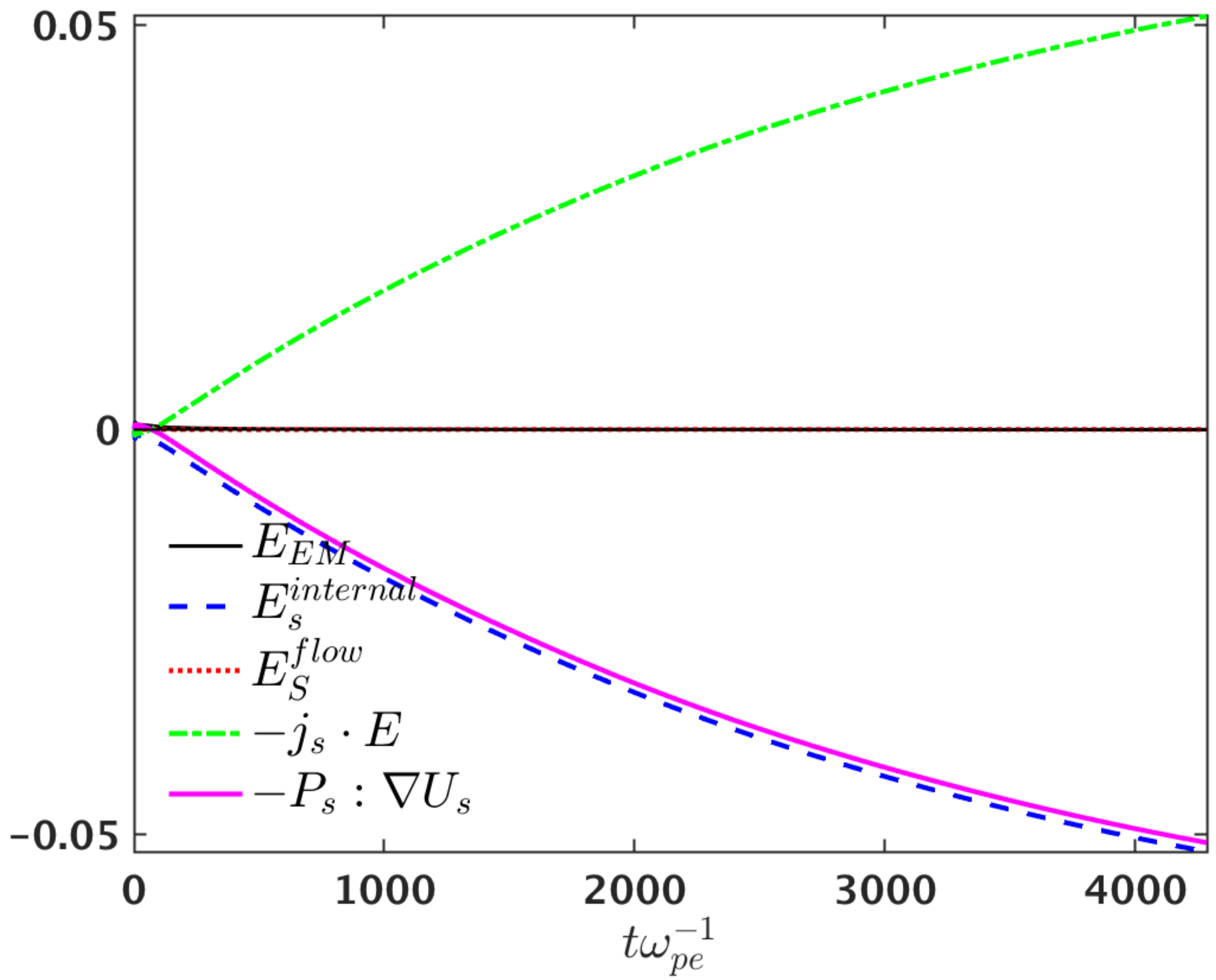

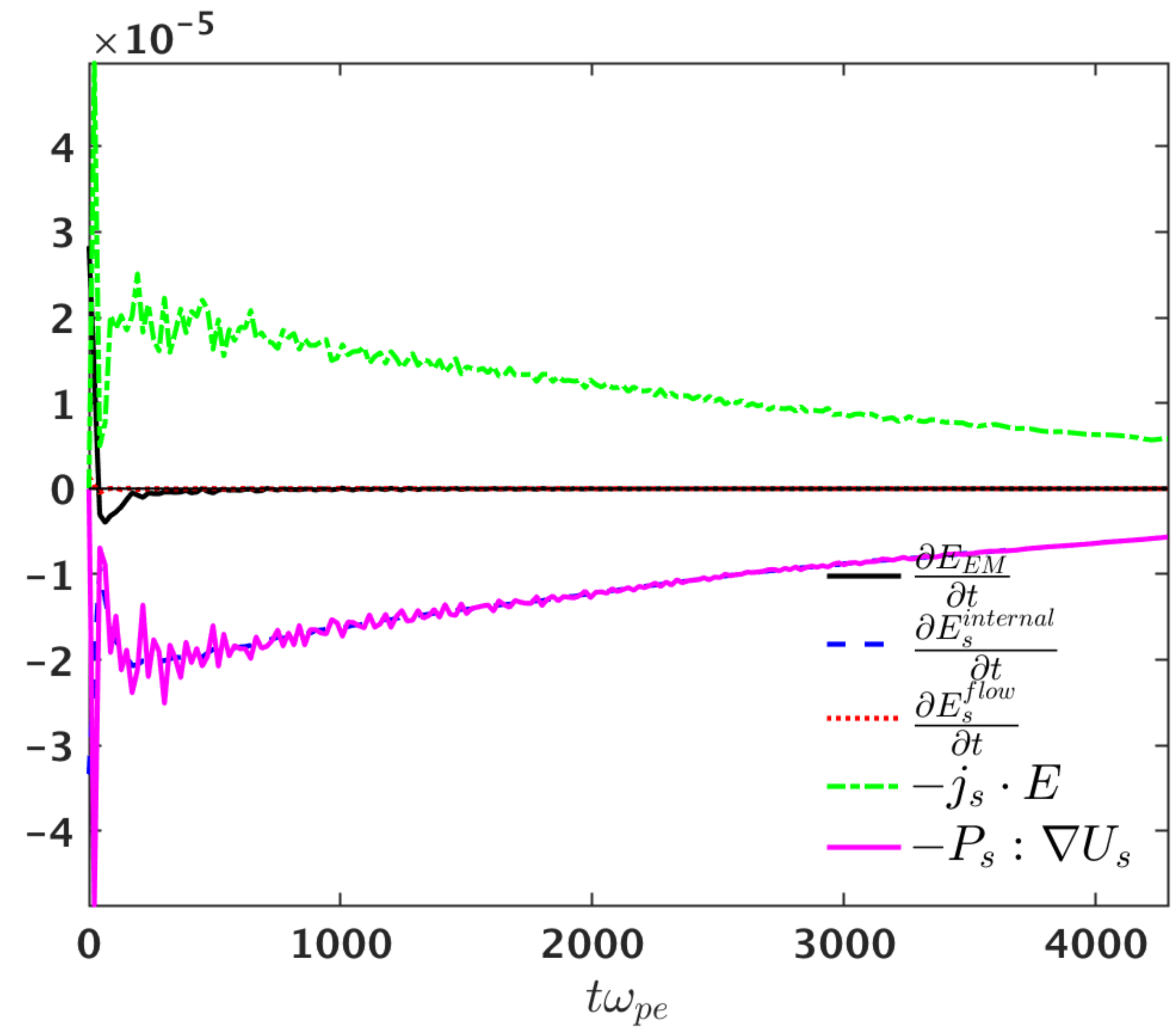

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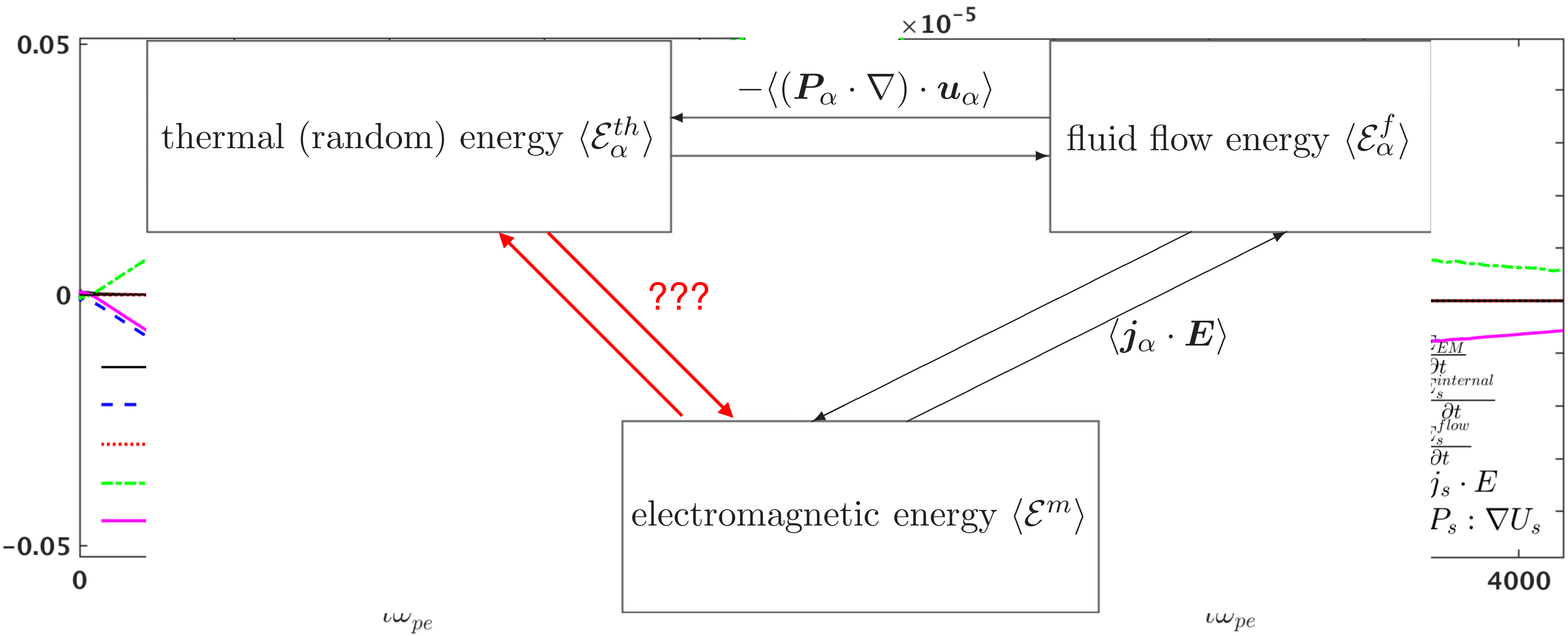
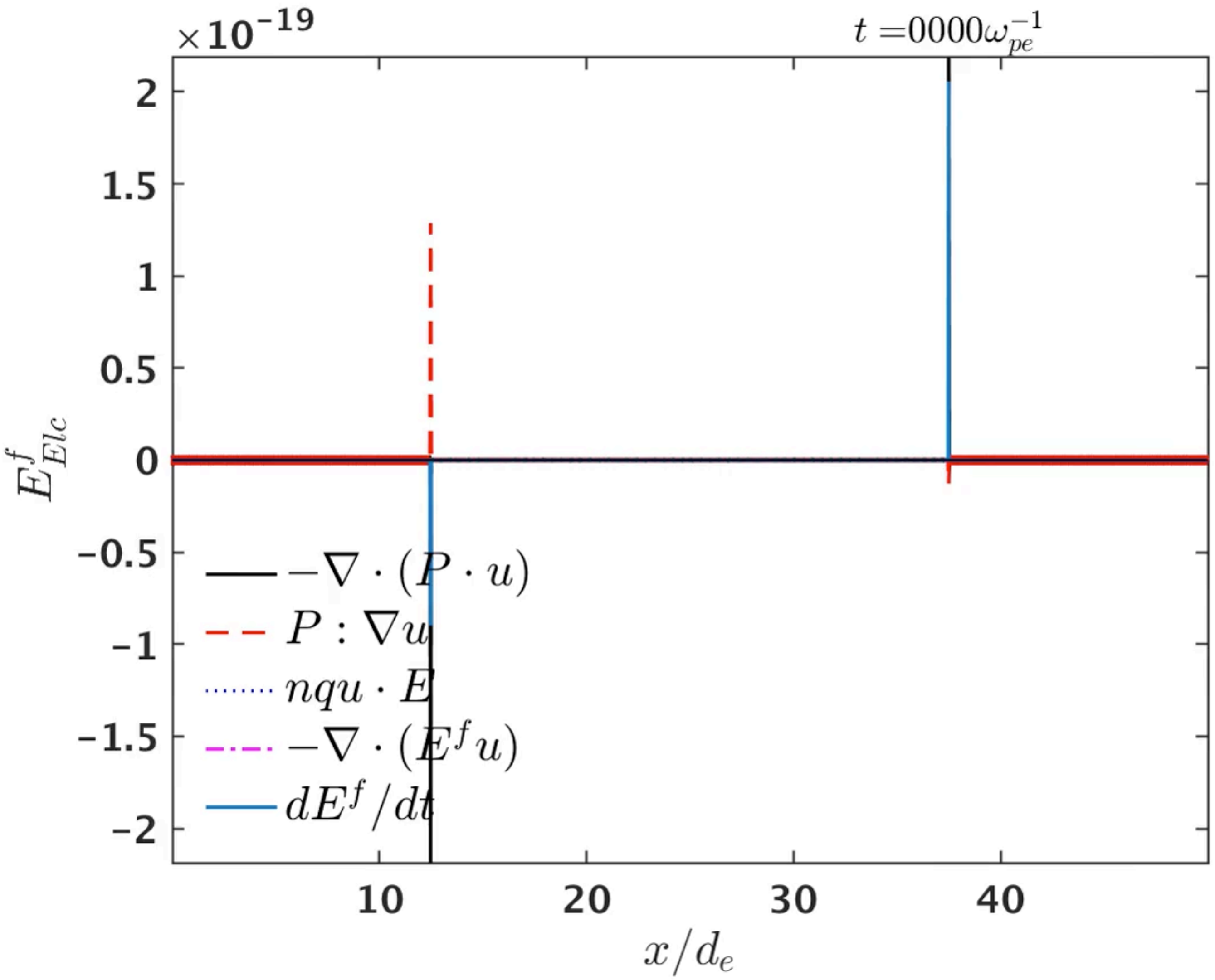
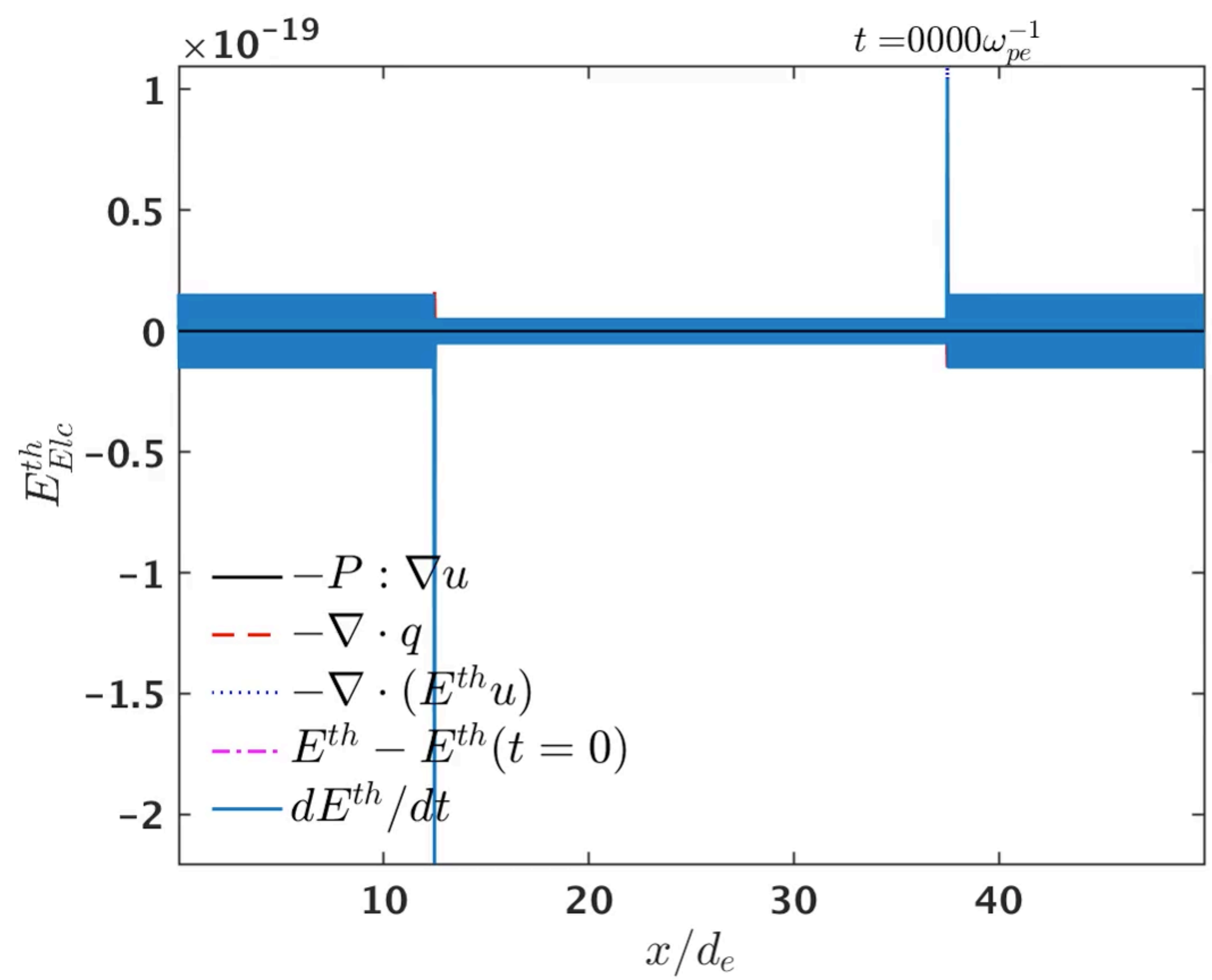


Sod shock spatial dependence electrons



$$\frac{\partial E^f}{\partial t} = -\nabla \cdot (uE^f) - \nabla \cdot (P \cdot u) + P : \nabla u + nqu \cdot E$$



$$\frac{\partial E^{th}}{\partial t} = -\nabla \cdot (uE^{th}) - \nabla \cdot q - P : \nabla u$$

Sod shock spatially integrated ions

$$\partial_t \langle \mathcal{E}^f_\alpha \rangle = \langle (\boldsymbol{P}_\alpha \cdot \nabla) \cdot \boldsymbol{u}_\alpha \rangle + \langle n_\alpha q_\alpha \boldsymbol{E} \cdot \boldsymbol{u}_\alpha \rangle,$$

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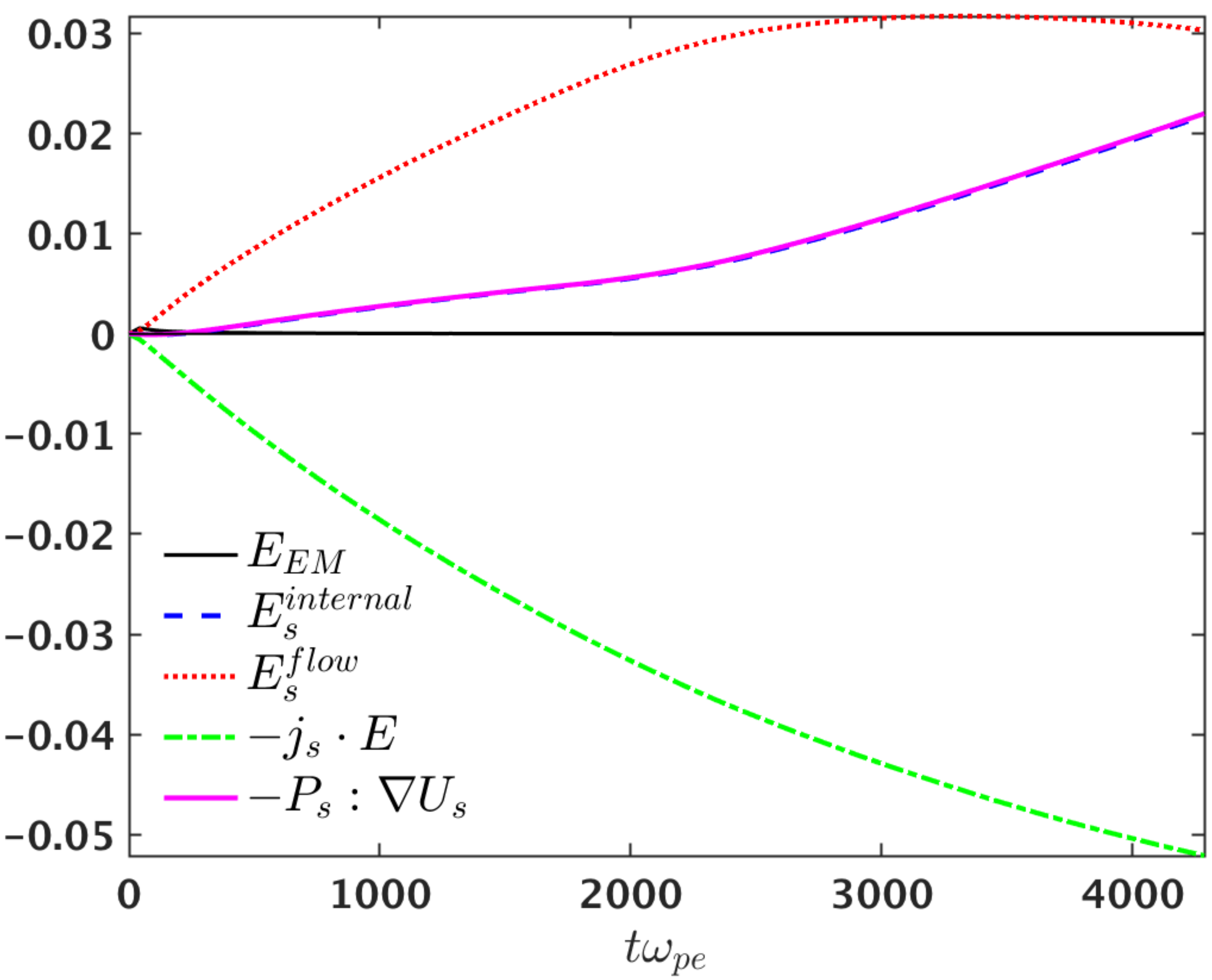
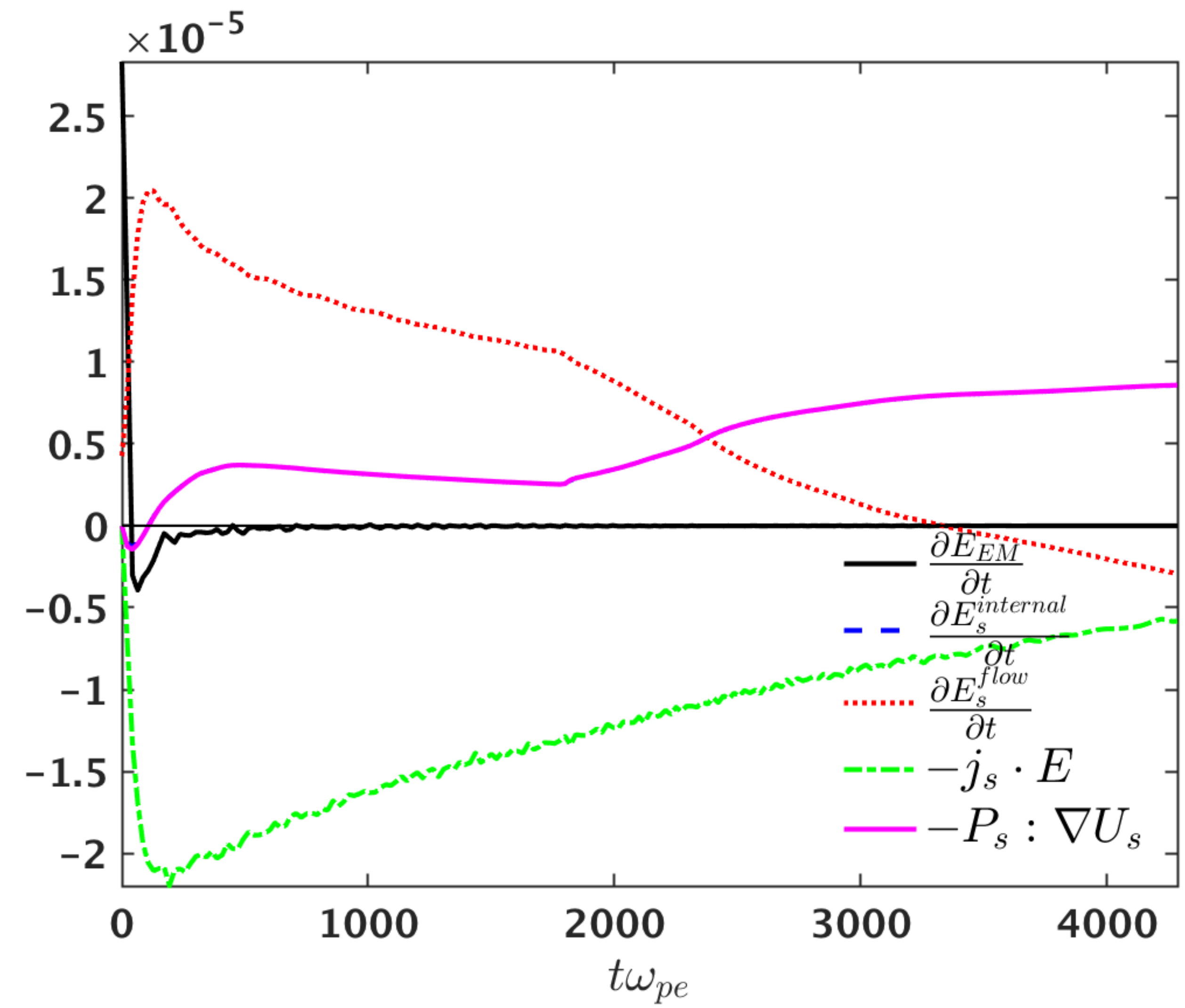
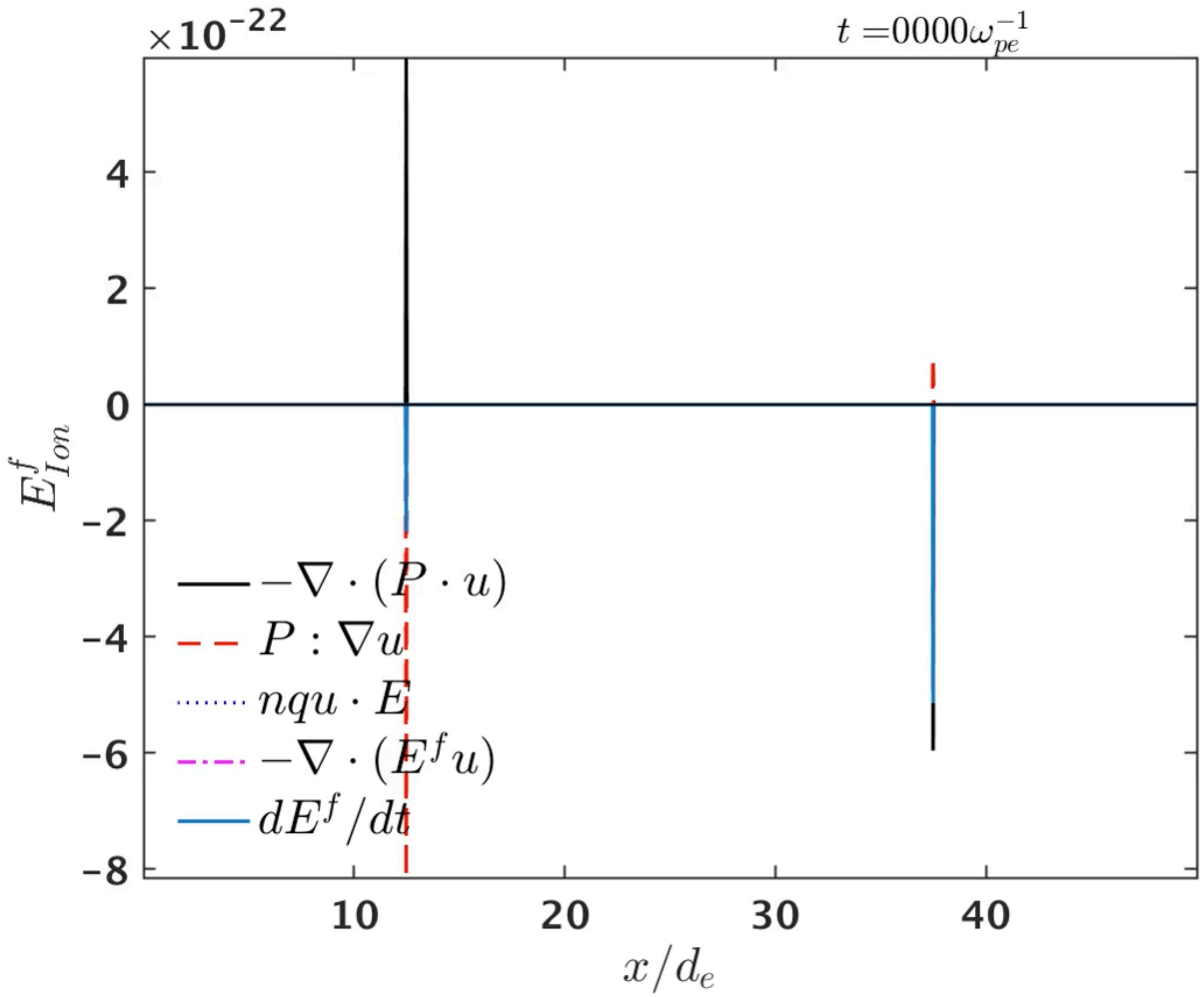




Sod shock spatial dependence ions



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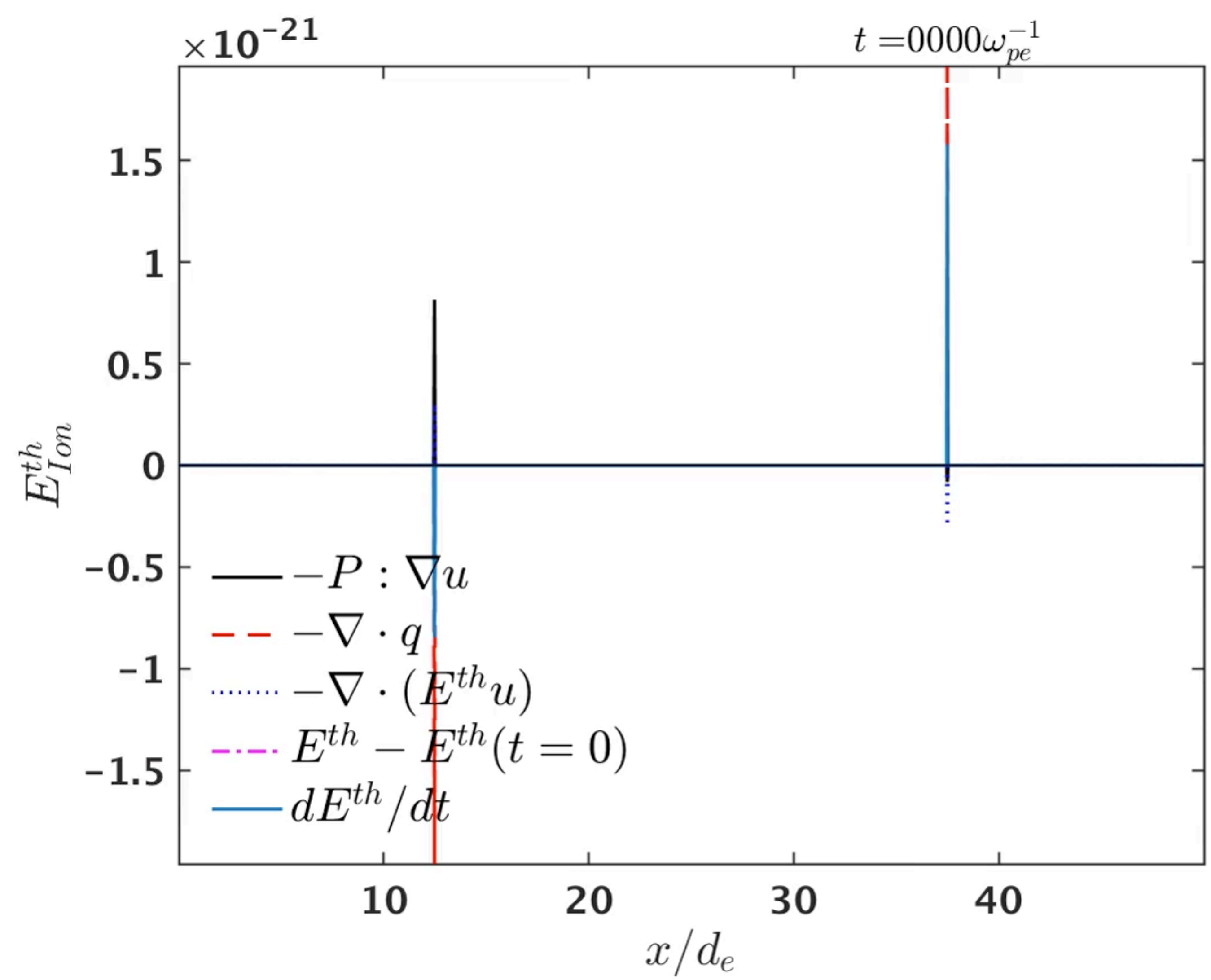

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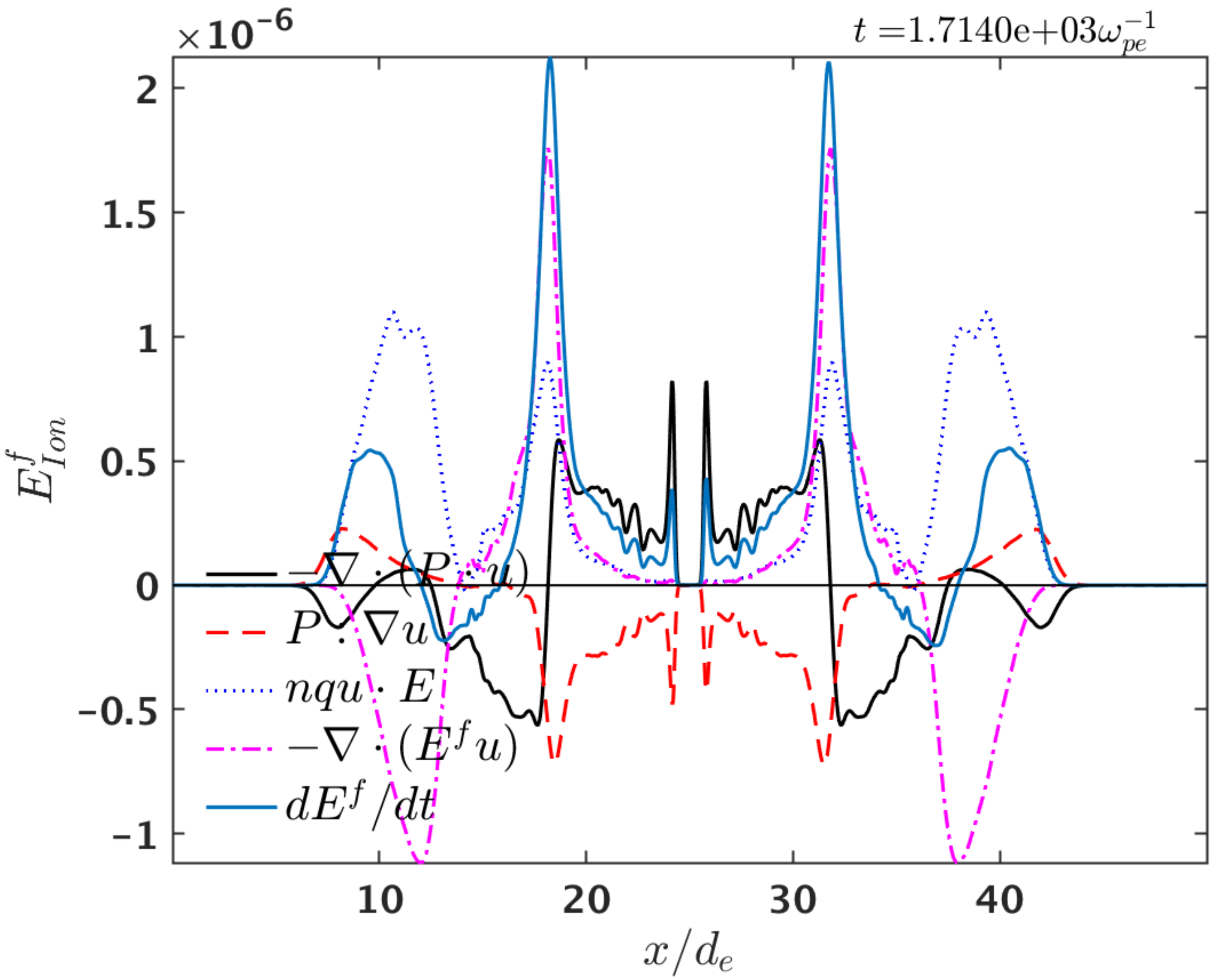
Sod shock spatial dependence ions



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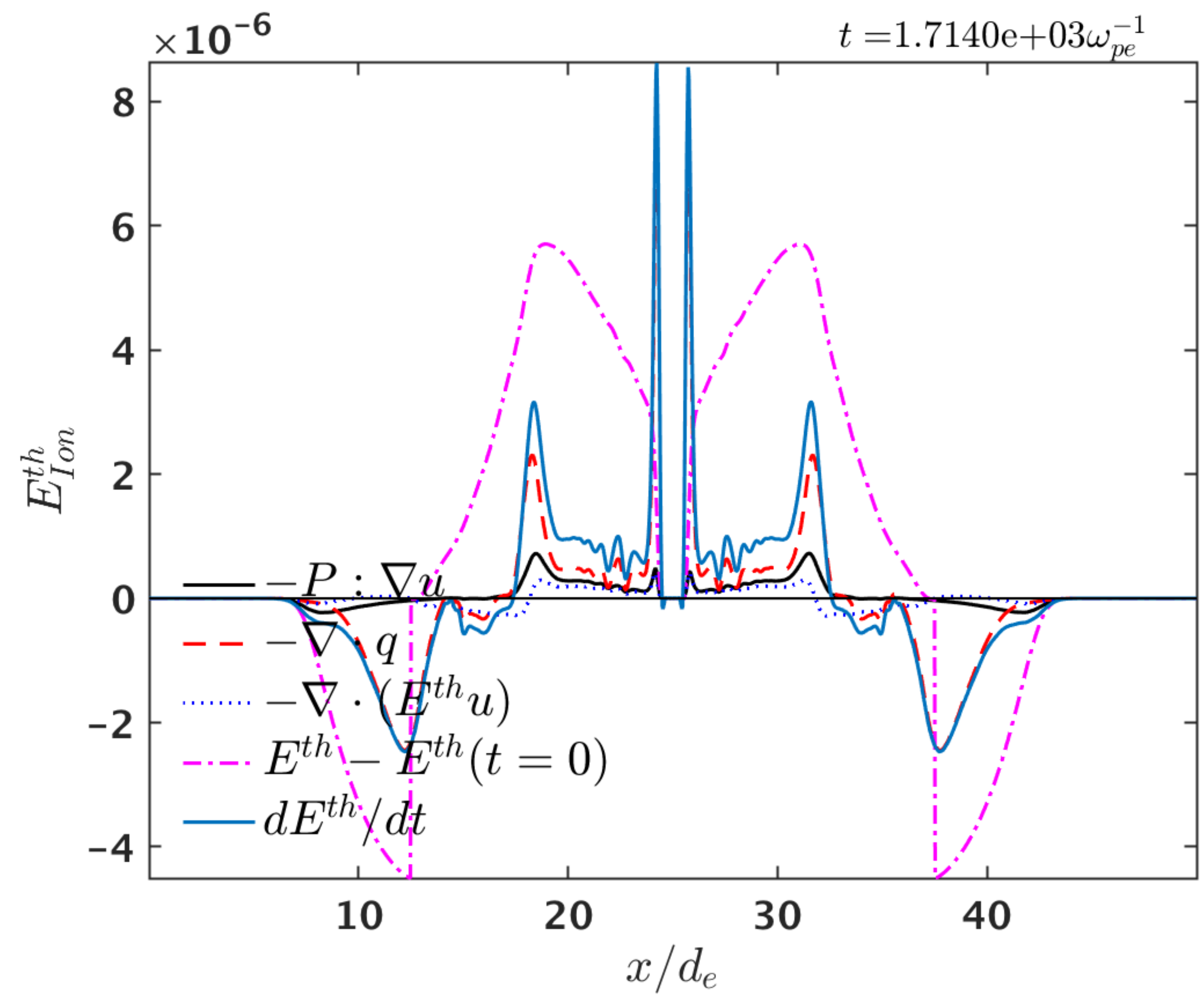

$$\frac{\partial E^{th}}{\partial t} = -\nabla \cdot (uE^{th}) - \nabla \cdot q - P : \nabla u$$

Strengths and weaknesses of PiD

- Agrees with total heating when integrated over entire domain
- Much like the FPC, it can be deceptive locally
- Provides little direct insight into mechanism(s) responsible for energy exchange
- Electromagnetic energy can be transferred directly to thermal energy
- In a collisional system (even weakly collisional), additional terms contribute to energy exchange

Conclusions/Future Work

- The FPC provides a relatively simply diagnostic to *identify the mechanism(s)* responsible for exchanging energy between the fields and particles
- The nature of the diagnostic permits its *use in single point spacecraft* data analysis
- In some cases, the field-particle term is *not the dominant local source* of particle kinetic energy
- The PiD diagnostic directly correlates with the gain or loss of thermal energy in a *collisionless* system
- The PiD diagnostic also provides insight into the location but *not the mechanism* of energy exchange and it suffers similar difficulty as the FPC when used locally
- Neither diagnostic takes collisions into account to determine irreversible heating

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- Both diagnostics show promise and provide useful but different insights